Holography

- History
  - Gabor 1948 → microscopy, decker
  - Heilman extended theory
  - 1960 - lasers → Leith, Demisuk
  - Gabor + Lippmann (French)

Wavestruct reconstruction problem

- Photo vs Holography (story)

- Recording - amp. 2 phase.

1st step: diff

2nd step: measure


4th step: wave

\[ a(x,y) = |a(x,y)| \exp[i \phi(x,y)] \]
\[ A(x,y) = |A(x,y)| \exp[i \Psi(x,y)] \]

\[ E(x,y) = |A(x,y)|^2 + |a(x,y)|^2 + 2|A(x,y)||a(x,y)| \cos [\Psi(x,y) - \phi(x,y)] \]

linear → log E recording
Assuming $A(\mathbf{x}, \mathbf{y}) = A$, uniform, bias in $t_A$

$$t_A(x, y) = t_p + \beta \left( |s| + A^* a + A a^* \right)$$

bias $> 0$ - negative
bias $< 0$ - positive

Reconstruction by $B(x, y)$

$$B(x, y) = t_A(x, y) = t_p + \beta A a^* B + \beta' A B a +$$

$$\beta' A B a^* = -U_1 + U_2 + U_3 + U_4$$

Let $B = A$ (exact same!)

$$U_3 = \beta' |A|^2 a$$ - original wavefront

Let $B = A^*$

$$U_4 = \beta' |A|^2 a^*$$ - phase conjugate

$$U_3 a$$ or $$U_4 a^*$$ Real image

Linearity: so far linear! Then $t_A(x, y)$ if $NL \rightarrow$ we get harmonic...
Image Formation

Object (Obj)

Real

Conj

or the book with arbitrary point source

Virtual

Real image

Conj at
Fabor Hologram

If a bright transparency dark will not work!

\[ t(x_0, y_0) = t_0 + \Delta t(x_0, y_0) \]

\[ \Delta t < t_0 \]

Then:

\[ I(x, y) = |A + a(x, y)|^2 = |A|^2 + |a|^2 + A^*a + Aa^* \]

in our case \( A \to t_0 \)

\( a \to \Delta t(x_0, y_0) \)

Twin Images

\[ t_A = t_B + \beta |(1/2 + \beta A^*a + Aa^*)| \]

Reconstruction: B

\[ B \cdot A = B \cdot t_B + B \cdot \beta |(1/2 + \beta A^*a + B^*Aa)| \]

\( a(x, y) \ll A \to 2^{nd} \text{ term negligible} \)

1st dominant \( \ll B \)

- twin images
- opt axis
- separation
- strong light
Gabor's hologram limitations

1) If a is strong we have diff. term - photography + weak to - no
interference.

2) Overlapping images in the same diff. far apart carrier to separate

Solution: with Upatnieks

\[ \text{Rec. med.} \]

\[ \text{Obj. 20.} \]

\[ \text{Recording.} \]

\[ \text{Total field} = \text{Reference} \]

\[ U(x,y) = A \exp\left[-j2\pi dy\right] + a(x,y) \]

\[ d = \frac{\sin 2\theta}{\lambda} \]

\[ I(x,y) = |A|^2 + |a(x,y)|^2 + A^*a(x,y)e^{j2\pi dy} + Aa^*(x,y)e^{-j2\pi dy} \]

Or alternatively

\[ I(x,y) = |A|^2 + a(x,y)^2 + 2|A||a(x,y)|\cos(2\pi dy + \phi) \]
- plane wave is a $\lambda$ to $\lambda$
- gives a $\lambda$ to $\lambda$
- explain all terms where they...

$U = \frac{1}{2} \beta A e^{i2kx}$

$U_1 = 1.5$  
$U_2 = 1.5 A e^{i2kx}$  
$U_3 = \frac{1}{3} |19/r - x - 2\pi|  
$1/3  
$2/3  
$- 1/3  
$2\pi  
$- 2\pi  
$- 2\pi  
$- 2\pi  
$- 2\pi  
$- 2\pi$

Virtual
Q: Minimum Key Angle:

\[ G_1(f_x, f_y) = t_b \delta(x - f_x, y - f_y) \]

\[ G_2 = \beta \cdot G_0(f_x, f_y) \oplus G_0^*(f_x, f_y) \]

\[ G_3 = \beta' \cdot A \cdot G_0^*(f_x, f_y - \alpha) \]

\[ G_4 = \beta' \cdot A \cdot G_0^*(-f_x, -f_y - \alpha) \]

Let

\[ \alpha > 3B \]

\[ \sin 2\alpha \geq 3B \]

\[ 2B \min = \sin^{-1} 3B \]
Holography of 3-D Scenes

Recording

Point Source

Film

Reconstruction: Virtual

Real

Practice: Resolution of film.
Types:
- Fresnel, Fraunhofer, Fourier,
- Approx. FT,
- Lenseless FT

Why:
- 2 spherical waves at same λ → linear fringes
- it is equivalent spatial phases → still different in ft. setup.

Transmission & reflection. Hol

So far → transmission.
\[ \theta_g = \frac{2\pi}{K_i} = \frac{1}{2 \sin \left( \frac{2\theta}{2} \right)} \]

When \( 2\theta \approx 180^\circ \), \( \sin 90^\circ \approx 1 \)

\[ \theta_g \approx \frac{1}{2} \text{ in } 2\text{-direction} \]

Thick emulsion is needed

Holographic Stereogram

Stereo: Each eye sees a different object projected on a horizontal parallax.

Recording

Film

Ref. Slit

Record space variant help each for each perspective on film \( \rightarrow \) in Reconst.
We will get in each eye a stereo pair.

Rainbow Hologram

Steve Benton

- White light illumination.

2-step process:

1. First hologram -> make a hologram in monochromatic light.

2. Second step.
   - Reconstruct 1st by "anti-ref."
   - Reconstruct original object as real

- Narrow slit
- Horizontal slit -> This removes vertical parallax.
Next, record H2

Reconstruct H2

Obj

image of the slit used to reconstruct H1

If the light is white,

- Colors will diffract $ightarrow$ disperse
- Both obj & slit so we will get slits.

Eve pupil + slits will give a specific color

Tall eyes $\rightarrow$ R

Short $\rightarrow$ B
Image locations & Magnification

Next met plane waves but point sources.

\[(x_r, y_r, z_r)\]

\[(x_0, y_0, z_0)\]

\[r, z_0 < 0 \rightarrow 1,\]
\[(x_p, y_p, z_p)\]

\[r_p < 0 \rightarrow 2\]

We use spherical waves:

\[U(x, y) = A e^{-j \frac{\pi}{\lambda z_r} [(x-x_r)^2 + (y-y_r)^2]} +\]
\[+ a e^{j \frac{\pi}{\lambda z_0} [(x-x_0)^2 + (y-y_0)^2]}\]

\[J = |A|^2 + |a|^2 + Aa^* \exp\left\{-j \frac{\pi}{\lambda z_r} [(x-x_r)^2 + (y-y_r)^2]\right\} +\]
\[+ j \frac{\pi}{\lambda z_0} [(x-x_0)^2 + (y-y_0)^2]\]
\[+ A^* a \exp\left\{j \frac{\pi}{\lambda z_r} [(x-x_r)^2 + (y-y_r)^2] - j \frac{\pi}{\lambda z_0}\right\}\]
\( t_3 = \beta' A a^* \exp \left\{ -j \frac{\pi}{\lambda_1 z_0} \left[ (x-x_r)^2 + (y-y_r)^2 \right] + j \frac{\pi}{\lambda_1 z_0} \left[ (x-x_o)^2 + (y-y_o)^2 \right] \right\} \)

\( t_4 = \beta' A^* a \exp \left\{ j \frac{\pi}{\lambda_1 z_0} \left[ (x-x_r)^2 + (y-y_r)^2 \right] - j \frac{\pi}{\lambda_1 z_0} \left[ (x-x_o)^2 + (y-y_o)^2 \right] \right\} \)

Reconstructing spherical wave:

\( U_p(x,y) = t_3 B \exp \left\{ -j \frac{\pi}{2p} \left[ (x-x_p)^2 + (y-y_p)^2 \right] \right\} \)

and we get

\( U_3 = t_3 B \exp \left\{ -j \frac{\pi}{2p} \left[ (x-x_p)^2 + (y-y_p)^2 \right] \right\} \)

\( U_4 = t_4 U_p \)

We can look for a reconstructed spherical wave in the form

\( U_i(x,y) = K \exp \left\{ -j \frac{\pi}{\lambda_2 z_i} \left[ (x-x_i)^2 + (y-y_i)^2 \right] \right\} \)

and equate the corresponding terms in \( U_3 \) and \( U_4 \):

\( \frac{1}{2i} = \frac{1}{2p} + \frac{\lambda_2}{\lambda_1 z_r} + \frac{\lambda_2}{\lambda_1 z_o} \)

When \( z_i < 0 \) \( \rightarrow \) virtual image

When \( z_i > 0 \) \( \rightarrow \) real image

Example of \( U_3 \):
\[ V_3 = \beta \left( \frac{A_1 e^{-j \frac{2\pi}{\lambda_1 z} r}}{1 + \frac{x^2}{\lambda_1^2 z^2}} + \frac{j \lambda_1 z}{1 + \frac{x^2}{\lambda_1^2 z^2}} \right) \]

\[ = \kappa e^{-j \frac{\pi}{\lambda_2 z_2} \left( x^2 + y^2 \right)} \]

Take \( \left( ^2 \right) \to i e^{-j \frac{\pi}{\lambda_2 z_2} \left( x^2 + y^2 \right)} \]

\[ = e^{-j \frac{\pi}{\lambda_2 z_2} \left( x^2 + y^2 \right)} \]

\[ = e^{-j \frac{\pi}{\lambda_2 z_2} \left( x^2 + y^2 \right)} \]

\[ - j \frac{\pi}{\lambda_2 z_2} \left[ \frac{1}{\lambda_1 z_1} \frac{1}{\lambda_0 z_0} + \frac{1}{\lambda_2 z_2} \right] = - j \frac{\pi}{\lambda_2 z_2} \frac{1}{\lambda_2 z_2} \]

\[ \frac{1}{\lambda_2 z_2} = \frac{\lambda_2 z_2}{\lambda_1 z_1} = \frac{1}{\lambda_1 z_1} + \frac{1}{\lambda_2 z_2} \]

\[ \text{linear phases:} \]

\[ e^{-j \frac{\pi}{\lambda_2 z_2} \left[ 2 \pi z_2 r \right]} e^{j \frac{\pi}{\lambda_2 z_2} \left[ 2 \pi z_2 x_0 \right]} e^{-j \frac{\pi}{\lambda_2 z_2} \left[ 2 \pi z_2 x_0 \right]} = e^{-j \frac{\pi}{\lambda_2 z_2} \left[ 2 \pi z_2 x_0 \right]} \]

\[ e^{-j 2\pi z_2 \left[ x_r - \frac{x_0}{\lambda_1 z_1} + \frac{x_0}{\lambda_2 z_2} \right]} = e^{-j 2\pi z_2 \left[ x_r - \frac{x_0}{\lambda_1 z_1} + \frac{x_0}{\lambda_2 z_2} \right]} \]

and

\[ x_i = \left[ \frac{\lambda_2 z_2}{\lambda_1 z_1} x_r - \frac{\lambda_2 z_2}{\lambda_1 z_1} x_0 + \frac{\lambda_2 z_2}{\lambda_2 z_2} \right] = \left[ \frac{\lambda_2 z_2}{\lambda_1 z_1} x_r - \frac{\lambda_2 z_2}{\lambda_1 z_1} x_0 + \frac{\lambda_2 z_2}{\lambda_2 z_2} \right] \]

and so on!
Result:
\[ x_1 = \pm \frac{\lambda_2 \xi}{\lambda_1 \zeta_0} x_0 \pm \frac{\lambda_2 \xi}{\lambda_1 \zeta_0} x_\theta + \frac{2i}{\eta_2} x_p \]
\[ y_1 = \pm \frac{\lambda_2 \xi}{\lambda_1 \zeta_0} y_0 \pm \frac{\lambda_2 \xi}{\lambda_1 \zeta_0} x_\theta + \frac{2i}{\eta_2} x_p \]

Axial/Magnification

Transverse

\[ M_\perp = \left| \frac{\partial x'_1}{\partial x_0} \right| = \left| \frac{\partial y'_1}{\partial x_0} \right| = \frac{\lambda_2 \xi}{\lambda_1 \zeta_0} = \]

\[ = 1 - \frac{\zeta_0}{\zeta_2} + \frac{\lambda_1 \zeta_0}{\lambda_2 \eta_2} \]

\[ M_\perp = \left| \frac{\partial 2i}{\partial \zeta_0} \right| = \left| \frac{\partial}{\partial \zeta_0} \left( \frac{1}{\eta_2} \pm \frac{\lambda_2}{\lambda_1 \zeta_0} \right) \right| = \]

\[ = \frac{\lambda_1}{\lambda_2} M_\perp^2 \]

In general, \( M_\perp \neq M_\perp \) causing 3-D distortions.

If the hologram is magnified by \( M^2 \),

Then

\[ M_\perp = m \left| 1 - \frac{\zeta_0}{\zeta_2} + m^2 \frac{\lambda_1}{\lambda_2} \frac{\zeta_0}{\eta_2} \right|^{-1} \]

\[ M_\theta = \frac{\lambda_1}{\lambda_0} M_\perp \]

\[ M_\perp = \frac{\lambda_1}{\lambda_0} \]
\[
\frac{\partial}{\partial z_0} \frac{1}{x} = -\frac{1}{x^2} \cdot \frac{dx}{\partial z_0} = -\frac{1}{x^2} \left( -\frac{\lambda_2}{\lambda_1 z_0^2} \right) = \frac{\lambda_2}{\lambda_1} \left( \frac{1}{\frac{z_0}{z_0^*} \pm \frac{\lambda_2 z_0}{\lambda_1 z_0^*} - \frac{\lambda_2}{\lambda_1}} \right) = \frac{\lambda_2}{\lambda_1} \left( \frac{1}{\left( \frac{z_0}{z_0^*} \pm \frac{\lambda_2 z_0}{\lambda_1 z_0^*} + \frac{\lambda_2}{\lambda_1} \right)} \right) = \frac{\lambda_2}{\lambda_1} \left[ \frac{1}{\left( 1 - \frac{z_0}{z_0^*} \pm \frac{\lambda_1}{\lambda_2} \frac{z_0}{z_0^*} \right)^2} \frac{\lambda_2^2}{\lambda_1^2} \right] = \frac{\lambda_1}{\lambda_2} M_t^2
\]
Thick Holograms

We have seen $A = 0 \Rightarrow$ same idea.
We had

$$Q = \frac{2\pi \lambda \text{od}}{h A^2}$$

$q > 2\pi \Rightarrow$ thick grating $\Rightarrow$ fringe

Recording thick hologram

$\theta$
\[ U(r) = A e^{jkr} \]
\[ U_0(r) = A_0 e^{jko} \]
\[ I(r) = |A|^2 + |A_0|^2 + 2|A||A_0| \cos(kr - ko) \]

\( \phi \) - phase diff. between \( A \) and \( A_0 \)

\[ K = kr - ko \]
\[ K = \frac{2\pi n}{A} \quad \text{grating wavevector} \]
\[ \Lambda = \frac{2\pi}{|K|} = \frac{1}{2 \sin \theta} \]

\[ \frac{K}{2} = kr \cdot \sin \theta \]
\[ \text{or} \quad K = 2kr \sin \theta \]
\[ \frac{2\pi}{\Lambda} = 2 \frac{2\pi}{2 \sin \theta} \sin \theta \]
\[ \Delta \Lambda = \frac{1}{2 \sin \theta} \]

Reconstruct:

say construct int own thing silver mirrors
\[ \sin \theta = \pm \frac{1}{2d} \]

This is Bragg's condition.

All comparing with

\[ \lambda = \frac{1}{2 \sin \theta} \]

We deduce

\[ \theta = \frac{\pi}{4} \pm (\pi - \theta) \]

Or equivalently we have a cone of angles

\[ \theta \]

This is referred as to Bragg-degeneracy.

Fringe orientation:
Gratings of finite size

- always finite # of minars!
- use 3-D Fourier analysis!

\[ g(r) = \iiint f(k) e^{i k \cdot r} \, dk \]

\[ d^3k = dk_x \, dk_y \, dk_z \]

"Let's consider simple volume cos. grating"

\[ g(r) = \left[ 1 + m \cos (K_\theta \cdot r + \phi_0) \right] \cdot \text{rect}(x/x) \cdot \text{rect}(y/y) \cdot \text{rect}(z/z) \]

\[ \phi_0 \text{ - unimportant fixed phase.} \]

\[ C(K) = \left[ \delta(K) + \frac{1}{2} \delta(K-K_\theta) + \frac{1}{2} \delta(K+K_\theta) \right] \]

\[ \times \text{sinc} \frac{xK_x}{2\pi} \, \text{sinc} \frac{yK_y}{2\pi} \, \text{sinc} \frac{zK_z}{2\pi} \]

This result \( \rightarrow \) blurs the tip of \( K_\theta \) into a "blur" \( \rightarrow \) meaning Bragg gets more possibilities.

Few examples.
More rigorous $\rightarrow$ Coupled Wave analysis.

\[ \text{Kogelnik} \]
Analysis

\[ \nabla^2 U + k^2 U = 0 \]

For a grating that has absorption:

\[ k = \frac{\omega}{c n} + jk \]

Absorption coefficient.

We assume within the grating, the \( n \) and \( d \) are:

\[ n = n_0 + n_1 \cos K \cdot r \]
\[ d = d_0 + d_1 \cos K \cdot r \]

\[ r = (x, y, z) \], \( K \) - grating wavenumber.

The grating is assumed to be:

The grating is assumed to be:

\[ n_x \]
\[ z \]
\[ \text{slant angle} \]

Assume:

i) Thin → only 1 wave is reconstructed!
\[ U_p(r) \rightarrow \text{reconstruction} \rightarrow \text{depleted \\& absorbed} \]

\[ U_i(r) \rightarrow \text{reconstructed} \]

So we assume total field in the volume is

\[ U(r) = U_p(r) + U_i(r) = \]

\[ = R(z) e^{-j \frac{\lambda_0 r}{k_0}} + S(z) e^{-j \frac{\lambda_0 r}{k_0}} \]

\[ \Rightarrow \text{replace } k_0 \text{ with } k_i (\text{following } \text{paper 22}) \]

\[ \frac{\Delta}{\lambda_0} = \frac{\Delta}{k_0} - \text{play back phase} \]

\[ \text{match Bragg cond} \]

2) We assume on distance 1, absorb \( \Rightarrow 0 \)

\[ n_0 k_0 >> \lambda_0 \]

\[ n_0 k_0 >> \lambda_0 \]

\[ n_0 >> n_i \]

Where \( k_0 = \frac{2 \pi}{\lambda_0} \) \( \rightarrow \text{this is } \text{modulation is small} \)

We rewrite

\[ k^2 = \left[ k_0 (n_0 + n, \cos K \cdot r) + j \left( \lambda_0 + 2 \cos K \cdot r \right) \right] \]

\[ \approx B^2 + 2j B \lambda_0 + 4 \pi B \cos K \cdot r \]

\[ B^0 = k_0 n_0; \quad \Delta = \frac{1}{2} (k_0 n_0, + j k_0) \text{ from} \]
Next step and use \( \text{UK}\frac{\partial \Phi}{\partial x} \) into the wave eq. We assume \( S(z) & R(z) - \) slowly varying getting harmonic terms match! We get using \( \sigma = p - K \)

\[
\sigma - K = p - 2K
\]

\[
p + K = \sigma + 2K
\]

Resulting eqs:

\[
CR \frac{dB}{dz} + \alpha_0 R = j2\pi S'
\]

\[
cS \frac{dS}{dz} + (\alpha_0 - j\pi) S' = i2\pi R
\]
Detuning parameter:
\[ \frac{\beta}{\sqrt{2}} - \frac{\left| \beta \right|^2}{2\beta} \]

\[ C_R = \frac{p}{\sqrt{2}} = \cos \theta \]
\[ C_S = \frac{\sqrt{2}}{\beta} = \cos (\theta - 2\chi) \]

Bragg mismatch (match: \( \chi = p - k \))

\[ B^2 - \left| \beta \right|^2 = B^2 - (p - k) \cdot (p - k) = \]
\[ = B^2 - \left| \beta \right|^2 + 2p \cdot k - k^2 = \]
\[ = 2g \cdot k \cos (4 + \frac{\pi}{2} - \theta) - k^2 = \]
\[ = 2g \cdot k \cdot \sin (4 + \frac{\pi}{2}) \cdot 2g \cdot k \sin (\theta - 4) - k \]

\( k = |k|; \quad p = |\beta| = B = k_0 n_0 \]

and
\[ \frac{\beta}{\sqrt{2}} - \frac{\left| \beta \right|^2}{2\beta} = k \left[ \sin (\theta - 4) - \frac{k}{2B} \right] \]

Note: \( \chi = 0 \) when \( \chi = p - k \) - Bragg matched.
Consider Bragg detuning:
\[ \theta = \theta_B - \Delta \theta - \text{small mismatch in angle} \]
\[ \lambda' = \lambda - \Delta \lambda \]

Sub in last eq:
\[ \frac{\theta}{\lambda} = K \left[ \Delta \theta \cos(\theta_B - \theta) - \frac{\Delta \lambda}{2\lambda} \right] \]

Using \( K = \frac{2\pi}{\lambda} \); \( h = \frac{2\pi}{\lambda} \)

In \( \lambda \to \) selectivity is best when \( \lambda - \text{sm} \)
in Refl. grating

Max. selectivity in \( \theta \) is when \( \Delta \theta \) is large.

Objects are separated by \( 90^\circ \) (since \( K < \sin \theta \))

Combination — use eq.

Back to coupled wave eqs:
\( R \)-increase from S \& vise versa

Next solutions: Unstapped \( y = 0 \)

\( y = 90^\circ \) Refl.
Solution for Thick Phase Transm.

1) \( \omega = d_1 = 0 \)

\[ \frac{dR}{dt} = j \omega S \]

2) \( c_s \frac{ds}{dt} - j \frac{d}{db} S = j \omega R \)

with boundary cond. \( R(0) = 1 \)
\( S(0) = 0 \)

Solution @ \( z = d \):

\[ S(d) = j e^{j \chi} \frac{\sin \left( \frac{\Phi}{\sqrt{1 + \frac{\chi^2}{\Phi^2}}} \right)}{\sqrt{1 + \frac{\chi^2}{\Phi^2}}} \]

where

\[ \Phi = \frac{\pi n d}{\lambda C_B} \]

\[ \chi = \frac{3d}{2C_B} = \frac{Kd}{2C_B} \left[ \Delta \Theta \cos(\Theta - \Psi) - \frac{d}{2\lambda} \right] \]

Diffraction efficiency

\[ \eta = \frac{|S(d)|^2}{|R(0)|^2} = \frac{\sin^2 \left( \frac{\Phi}{\sqrt{1 + \frac{\chi^2}{\Phi^2}}} \right)}{1 + \frac{\chi^2}{\Phi^2}} \]

when illuminated at \( \beta = 45^\circ \)
\( \chi = 0 \) and
\( \eta = \sin^2 \Phi \)
The diff. eff. is \[ \frac{R_f}{R(0) \text{R}} \]

- No loss
- Power couples back & forth
- max @ \( \phi = \frac{\lambda}{2} = \frac{\pi n d}{\lambda} \)
- When \( \theta + \alpha \to \theta \text{ Bragg} \), it is diff. int.

Solution for a thick Amplitude Grating

Same as before but \( n_i = \) only \( \lambda_0, \lambda_i \)

\[ S(x) = -\exp\left(-\frac{x \frac{d \lambda}{2 \omega}}{\cosh(\frac{\phi}{\alpha})} \right) e^{jx \sinh(\frac{\phi}{\alpha})} \]

\[ \phi_d = \frac{\lambda \frac{d \lambda}{2 \omega}}{\cosh(\frac{\phi}{\alpha})} \]

\[ X = \frac{\phi_d}{2 \omega} \]

an Bragg \((x_d, \phi) = 0\)

\[ \eta = \exp\left(-\frac{2d \phi}{\cosh(\frac{\phi}{\alpha})} \right) \sinh\left(\frac{\phi_d}{2 \omega} \right) \]
1st term - absorption on \( \frac{1}{\cos \theta} \)

2nd arising effect due to diffraction

\( \lambda_1 \leq \lambda_0 \) - modulus

max. \( \eta \) when \( \lambda_1 = \lambda_0 \)

\( \Phi' = \frac{\lambda_0 d}{\pi \lambda} \)

\( \eta_B = \exp(-\pi \Phi' \lambda) \sinh^2(\Phi' \lambda) \)

Solution for thick reflection grat

\( \gamma = 90^\circ \)

\( R(0) = 1, \ S(d) = 0 \)

\( S(0) = -j \left[ -i \frac{\chi}{\Phi} + \sqrt{1 - \frac{\chi^2}{\Phi^2}} \coth \Phi \sqrt{1 - \frac{\chi^2}{\Phi^2}} \right]^{-1} \)

\( \chi, \Phi \) - defined before.
\[ \eta = \left[ 1 + \frac{1 - \frac{x^2}{\Phi^2}}{\sinh^2 \left( \Phi \sqrt{1 - \frac{x^2}{\Phi^2}} \right)} \right]^{-1} \]

\( \Omega \) : \text{Approx. } \chi = 0

\[ \eta_B = \tanh^2 \Phi \]

\[ \eta_B \approx 10 \]

Mismatch at \( \Phi = 6 \)

Thick Amplitude Reflection

Grating

\[ n = 0 \]

\[ S(\theta) = -j \left[ -j \frac{x a}{\Phi a} + \sqrt{1 - \frac{x a^2}{\Phi a^2}} \cosh \left( \Phi \sqrt{1 - \frac{x a^2}{\Phi a^2}} \right) \right] \]

where \( \Phi a = \frac{x d}{2 \cos \theta} \) and \( x a = \frac{x d}{2 \cos \theta} + \frac{j x d}{2 \cos \theta} \)
Under Bragg matching

\[ \frac{\varphi}{2} = 0 \]
and \( \eta_B \to \text{max} \) when \( \lambda_1 = \lambda_0 \)
under this cond:

\[ \frac{\chi_a}{\Phi_a} = 2 \quad \text{and} \quad \eta_B = \left[ 2 + \sqrt{3} \coth (\sqrt{3} \Phi_a) \right]^{-2} \]

\[ \eta_B + 0.07 \]

When \( \frac{\varphi}{2} \neq 0 + \varphi_l = \lambda_0 \)
\[ \chi_a = 2\Phi_a + j\chi \]
with \( \chi = \frac{Kd}{2\cos \Theta} \left( \Delta \Theta \cos(\Theta - 4) - \frac{\Delta l}{2\pi} \right) \)

\[ \eta_B = \frac{1}{2} + j \frac{\chi}{\Phi_a} + \sqrt{\left(2 + j \frac{\chi}{\Phi_a} \right)^2 - 1} \coth \left( \Phi_a \right) \]

not much selectivity


\[ \rightarrow 100\% \quad 3.7\% \quad 1000 \quad 7.2\% \]
9.8 Recording Materials:

1. Silver Halide Emulsion
   - High resolution: 2000 – 7000 cycles/mm (8E75 A/m)
     * conventional photo ~ 200
   - Spectral sensitivity (nm) < 700 (599 F)
   - Sensitivity: 10 - 100 mJ/cm²
   - Thickness of emulsion: 6 - 17 pm.
     (699 F - 17 pm)

2. Photopolymer film
   - Phase holograms – low insertion loss.
   - Thick (up to 8 mm) – high efficiency

Effect → photopolymerization

1) Monomer → crosslinking under light
2) Monomer diffuses to exposed areas
   (away from high concentration)
   More exposure takes place
3) Uniform exposure → fixing like
\[ \sin \theta \text{ is } \sin \frac{0.5}{\lambda} \to \text{ huge}\]

\[ \eta \propto \sin^2 \frac{\theta}{2} \]

\[ \frac{\sin^2 \frac{\theta}{2}}{1} \approx 0.002 \cdot 1000 \mu m = \frac{2}{1 \text{ mm}} \to \frac{2}{1 \mu m} \]

Developing \to UV light

Few mJ/cm²

Few companies:

- Dyes
- Omniderm
- Polaroid

3. Dichromatic gelatin

\[ \eta \approx 90\% \text{, high exposure used } 649F \]

Many theories how but not finite.

4. P.R.

- LiNbO₃, BSO, B60, KTN, BaTiO₃

Describe the mechanism

"Coupling!"
\[ t_A(x,y) = \exp \left[ j 2\pi \frac{n(x,y) \Delta z}{\lambda_0} \right] \]

Weak: \(\Delta n \approx 10^{-4} - 10^{-5}\)

\[ t_A(x,y) = \exp \left[ j 2\pi \frac{\Delta n \Delta z \sin \frac{2\pi x}{\lambda_0}}{\lambda_0} \right] \approx \]

\[ \approx 1 + j 2\pi \frac{\Delta n \Delta z}{\lambda_0} \sin \frac{2\pi x}{\lambda_0} \]

\(\Delta n\): max amplitude; \(\lambda\): grating \(\frac{1}{2\sin \theta}\)

Explanation:
Degradation of Holographic Images.

- Effect of film size on imaging
  \[ M(f_x, f_y) = P(x, y) \]

  \[ \text{pupil} \rightarrow \text{size of film} \]

- Film resolution
- Film degradations \( ? \)

Effect of Film.
We used MTF in the past;
We will evaluate 2 geom:

Collimated Ref.

\[ U_r(x, y) = A e^{-j 2\pi y} \]
\[ U_0(x, y) = a \exp \left[ -j 2\pi \left( f_x x + f_y y \right) \right] \]

\[ I(x, y) = A^2 + |a|^2 + 2A|a| \cos \left\{ 2\pi \left[ f_x x + (f_y y) \right] \right\} \]

\[ \phi = \arg (a) \]

Let emulsion have MTF \( \Rightarrow M(f_x, f_y) \)
Then effective exposing intensity \( I_{eff} \) is

\[ I_{eff} = A^2 + |a|^2 + 2A|a| M(f_x, f_y) \cos \left\{ \ldots \right\} \]

Weighing coeff. of spot size.
Graphically, the object spectrum is centered:

\[ \text{MTF is centered} \quad \text{at} \quad f_y = \lambda \]

Image spectrum:

\[ f_y = \lambda \]

1) \[ H(f_x, f_y) = M(f_x, f_y - \lambda) \]

filtered image with transfer function

2) \[ P(x, y) = \begin{cases} 1 & \text{in hole} \\ 0 & \text{otherwise} \end{cases} \]

Then

\[ H(f_x, f_y) = P \left( \frac{x}{\lambda} \right) \cdot M(f_x, f_y - \lambda) \]

equivalently, scale in space!

\[ p(x, y) = M \left( \frac{x}{\lambda} \left( \frac{x^2}{\lambda^2} - 1 \right), \frac{y}{\lambda^2} \right) \]

effective transparency in the pupil plane
For both
\[ I(x,y) = A^2 + |a|^2 + 2A|a| \cos \left[ 2\pi \left( \frac{x-x_0}{\lambda/2} \right) + \phi \right] + 2\pi \left( \frac{y-y_0}{\lambda/2} \right) + \phi \]

In other words both geometries:

- angle encoded as spatial frequency of interference fringes encoded.

\[ f_x = \frac{x_0 - x_r}{\lambda/2} \]
\[ f_y = \frac{y_0 - y_r}{\lambda/2} \]

Effect of MTF? Same:

\[ I_{\text{eff}} = A^2 + |a|^2 + 2A|a| M\left( \frac{x_0 - x_r}{\lambda/2}, \frac{y_0 - y_r}{\lambda/2} \right) \cos \left[ \ldots \right] \]

Fringes are weighted by value of MTF. However here the weight affects the object (image) since it modulates the cos corresponding to object points relative to net restricts field of view about center.
affects the spectrum of obj.

affects the extent of object (size of the object) in relation to ref.

Effect of film nonlinearity

- Thin hologram.

- Then you get orders! Like in grading J/L

- More complete → literature

Effect of grains in film

- Scattering → reduces contrast of received
- Enhancing detection sensitivity due to gain of strong ref. beam!
Speckle noise

\[ \Delta x = \frac{\lambda z}{D} \] size of hologram.

Applications of Holography

- Microscopy: X-ray (electron) holography, active res. area.
- Go to UV, EUV, VUV, etc.
- Attosec. optics in 100eV \( \lambda \) etc.
- Transverse res.: \( \frac{\lambda}{NA} \)
- Longitudinal: \( \frac{\lambda}{(NA)^2} \) \( \lim_{NA \to 0} \)

- Interferometry: multiple exposure = coherent addition of complex wavefronts

\[ E = \sum \frac{E_k T_k}{t_k}, \quad T_i = \text{exp. time} \]

Then for same reference! (How about changing the reference?)
\[ E = \sum_{k} \frac{\text{Tr} A_{k}^{2}}{k} + \sum_{k} \frac{\text{Tr} A_{k}^{*} A_{k}}{k} + \sum_{k} \text{Tr} A_{k} A_{k}^{*} + \sum_{k} \text{Tr} A_{k} A_{k}^{*} \]

For linear recording terms of interest

\[ t_{2} = \beta \sum_{k=0}^{N} \text{Tr} A_{k} A_{k}^{*} \]

\[ t_{\beta} = \beta \sum_{k=1}^{N} \text{Tr} A_{k} A_{k}^{*} \]

The terms have superposition of \( a_{1}, a_{2}, a_{3}, \ldots \)

Can be seen as interferometry with multiple beams!

- Double exposure \( \rightarrow \) fringes show phase difference
  
  **Real time**

- Make a hole
- Insert back
  - Perturb the obj
  - You get interference
Vibration analysis

Let $\mathbf{b}_i$ vibrate at $z_2$, with amplitude $m(x_0, y_0)$ with phase vibration $m(x_0, y_0)$

The light incident at hologram recording plane in point $(x, y)$ has phase

$$
\phi(x, y; t) = \frac{2\pi}{\lambda} \left( \cos \theta_1 + \cos \theta_2 \right) m(x_0, y_0)
$$

Using thin phase element we get

$$
F(r) = \sum \frac{1}{r} \exp \left[ i \phi(x_0, y_0) \right] = \sum \frac{J_0 \left( \frac{2\pi}{\lambda} \sqrt{\frac{m(x_0, y_0)}{\lambda}} \right) \cos \left( \frac{2\pi}{\lambda} \sqrt{\frac{m(x_0, y_0)}{\lambda}} \right)}{2}.
$$
Assume exposure time $t \gg \frac{2\pi}{\Omega}$

Then the "fringes" in time will wash out and the only hologram will be recorded at $k=0$, i.e., $S(y)$

All other terms will be $0$

\[ I_0 \left[ \frac{2\pi}{\lambda} \left( \cos\Theta_1 + \cos\Theta_2 \right) m(x_0, y_0) \right] \]

Intensity $\propto I_0$ and in each $(x_0, y_0)$ assuming $\Theta_1$ & $\Theta_2$ - same we get

IFF. value $I_0 \left( m(x_0, y_0) \right)$ i.e. modulated in intensity

Imaging through distinct medium

Obj. \quad Dist. \quad Medium \quad Ref. Hol. \quad Rec.

Image.

\[ U_0 \left( \frac{3}{2} \right) e^{-j} \]

Reconst.

\[ U_0^* \]

\[ = U_0^* \]
\[ T = \sum \left( \frac{1}{4} \lambda_1^2 + \lambda_2^2 + \lambda_3^2 + \lambda_4^2 + \lambda_5^2 \right) \]

I \left( x^p, y^p, \lambda \right) = \text{unknown}