1. Prove the following Fourier Transform relations:

   b) \( \mathcal{F}\{\text{sgn}(x)\text{sgn}(y)\} = \left( \frac{1}{j\pi v_x} \right) \left( \frac{1}{j\pi v_y} \right) \)

   c) \( \mathcal{F}\mathcal{F}\{f(x,y)\} = \mathcal{F}^{-1}\mathcal{F}^{-1}\{f(x,y)\} = f(-x,-y) \)

   d) \( \mathcal{F}\{f(x,y)h(x,y)\} = \mathcal{F}\{f(x,y)\}*\mathcal{F}\{h(x,y)\} \)

   e) \( \mathcal{F}\{\nabla^2 f(x,y)\} = -4\pi^2 (v_x^2 + v_y^2) \mathcal{F}\{f(x,y)\} \)

   where \( \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \) is Laplasian operator

Consider in b)

\[ \text{sgn}(x) = \lim_{N \to \infty} g_N(x) \]

where

\[ g_N(x) = \begin{cases} \exp(-x/N) & x > 0 \\ -\exp(x/N) & x < 0 \\ 0 & x = 0 \end{cases} \]

2. Suppose that a sinusoidal input

\[ f(x,y) = \cos[2\pi (v_x x + v_y y)] \]

is applied to a liner system. Under what (sufficient) conditions is the output a real sinusoidal function of the same spatial frequency as the input? Express the amplitude and phase of that output in terms of an appropriate characteristic of the system?