Problem 1: A gas laser emits a Gaussian beam waist from the output face, where the beam has wavelength 633nm, 1 mW total power, and a full spot width (2\(w_o\)) 100 microns in diameter.

(a) Determine the angular divergence of the beam in radians, the depth of focus in mm, and the beam diameter (full width) after it has propagated the approximately 3.5x10^5 km distance to the moon.

(b) What is the radius of curvature of the wavefunction at the output face (z=0), at z = \(z_0\), and at z=2\(z_0\)?

(c) What is the optical intensity (in W/cm²) at the beam center (x=y=z=0) and at the azial point z=\(z_0\)? Compare this with the intensity at z=\(z_0\) of a 100W spherical wave produced by a point-like isotropically emitting light source at z=0.

Problem 2: Assume that the radius of curvature and the width of a Gaussian beam of wavelength 1\(\mu\)m at some point on the beam axis are R1 = 1m and W1 = 1mm, respectively. Determine the beam width and radius of curvature a distance d = 10cm further along the beam axis.
1 Problem 1

We are given $\lambda = 633nm$, $P = 10^{-3}W$ and $W_0 = 0.05mm$

For problem a:

$$\theta_0 = \frac{\lambda}{\pi W_0} = 4.03 \times 10^{-3}\text{rad} = 0.231^\circ$$ (1)

$z_0 = \frac{W_0}{\theta_0} = 0.012m$ and depth of focus is $2z_0 = 0.025m = 2.5cm$

At $z = 3.5 \times 10^5km$, $W(z) = W_0[1 + (z/z_0)^2]^{1/2} = 1.41 \times 10^6 = 1410km$. Diameter is $2821km$.

For problem b:

At $z = 0$, $R = \infty$.
At $z = z_0$, $R = 2z_0 = 2.5cm$
At $z = 2z_0$, $R = z[1 + (z_0/z)^2] = 0.031m = 3.1cm$

For problem c:

At beam center, $I = I_0 = 2P/\pi W_0^2 = 2.546 \times 10^5W/m^2 = 25.46W/cm^2$.
On beam axis at $z = z_0$, $I = I_0[W_0/W(z_0)]^2 = I_0/2 = 12.73W/cm^2$
A spherical wave of power $P = 100W$ at $z = z_0 = 2.5cm$ has intensity

$$I = \frac{P}{4\pi z^2} = 5.169 \times 10^4W/m^2 = 5.169W/cm^2$$ (2)

The energy is $10^5$ times larger, its intensity is still 2 times less.
2 Problem 2

We can use the equation

\[
\frac{1}{q} = \frac{1}{R} - \frac{j}{\pi W^2} \tag{3}
\]

In problem 2, \( \lambda = 10^{-6}m \), at position 1, \( R_1 = 1m \) and \( W_1 = 10^{-3}m \). \( q_1 = 0.91 + j0.29 \)

At position 2, \( z_2 = z_1 + d \), where \( d = 0.1 \).
Therefore \( q_2 = 1.01 + j0.29 \) and \( 1/q_2 = 0.92 - j0.26 \). Based on \( q_2 \) we can calculate:

\[
R_2 = \frac{1}{0.92} = 1.09m \quad \frac{\lambda}{\pi W_2^2} = 0.26 \quad W_2 = 1.11 \times 10^{-3}m \tag{4}
\]