Problem 1: The paraxial approximations are that \( \sin(\theta) = \theta \) and \( \cos (\theta) = 1 \), where \( \theta \) is in radians and "small." Graph the percentage error (difference between real and approximate values) for each of these, and identify the angle (in degrees) where the error is 0.10%, 1% and 10%.

Problem 2: An arrow of height 20mm is located 150mm from a thin lens of focal length \( F = 60mm \), located at \( z=0 \) on the optical axis. A second arrow, of height 10mm, is located at \( z = -40mm \). (see Figure 1) The most elementary form of Gauss' lens law, for a thin lens in air, is \( 1 / f = (1 / l_o) + (1 / l_i) \), where \( l_o \) is the distance from the object to the lens (along the optic axis), and \( l_i \) is the distance from the lens to the image formed (positive values mean a real image forward from the lens on the optical axis). For each object, answer the following:

(A) Find the height and axial position of the image formed, measured from the lens face.
(B) Draw the system (to scale), showing the object, image, and paths of the chief and parallel rays.
(C) On the same drawing show the path of the marginal ray, hitting the bottom of the lens aperture.
(C) Calculate the transverse magnification, \( M_t = - l_o / l_i \).

Problem 3: Consider an object with axial extent, for example an arrow flipped to lie along the optical axis, with the base \( l_{o2} \) and tip \( l_{o1} \) from the lens. This object has positive length if \( l_{o2} > l_{o1} \). We can define the axial magnification \( M_a \) of the image formed by considering the image of the tip (at \( l_{i1} \)) and base (at \( l_{i2} \)) as \( M_a = (l_{i1} - l_{i2}) / (l_{o2} - l_{o1}) \), positive if the arrow's image points the same direction as the object.

(A) Find the axial magnification for the two short arrows shown in Figure 2.
(B) Express the axial magnification in terms of the transverse magnification of the object's end points.

Problem 4: What is the overall focal length of a pair of lenses, where the first and second lenses are focal length \( f_1 = 50mm \) and \( f_2 = -25mm \), and where the two lenses are separated by (A) 10mm, and (B) 25mm. (C) Does your answer to parts A or B change if the positions of the two lenses is reversed?

Problem 5: Introducing a glass block into an imaging system can change the position and size of the image formed. In the system of Figure 3, a paraxial 'thin' lens (in air) images through a glass block in contact with the lens. Note that the real image might be formed in air, or in the glass block.

Without the glass block in place (just the lens in air):

(A) Calculate the paraxial focal position (distance of image plane from lens) for image formed.
(B) What is the lateral magnification of the image formed?
(C) Sketch this system, to scale, showing the parallel ray, chief ray, and focal ray paths, and show the position and orientation of the image formed.

Now with the glass block in place, as shown below:

(D) Calculate the paraxial focal position (distance of image plane from lens) for the new image formed.
(E) What is the new lateral magnification of the image formed?
(F) Sketch the ray paths, showing the parallel ray, chief ray, and focal ray paths, and show the position and orientation of the new image formed.
Figure 1

Figure 2:

Figure 3:

Lens, \( f = 50 \text{ mm} \)
Glass block, \( n = 1.5 \)
focal position?