The exam is closed book, but one page of notes is allowed, as are calculators. Make sure you understand what is asked: read the entire question before starting. You must show your work and write (and draw!) legibly to get full credit. The solid angle enclosed by cone of half-angle $\theta$ is $2\pi[1-\cos(\theta)]$.

Name: SOLUTIONS

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Question 1: Telescope (10 points)

A telescope made of a 5cm focal length objective and 1cm focal length eyepiece separated by 6cm, as shown.

1) Sketch the beam path for a distant star located a 4° angle off axis, showing the path of the 5 incident rays, the exact height and position of the internal image, and the approximate position of the focal spot on the retina.

Internal image @ \( z = F_r = 5 \text{cm} \), height is \( F_r \tan \theta = 5 \text{cm} \tan 4° = 0.35 \text{cm} \)

2) Does a pattern of stars viewed through the telescope appear to the viewer to be erect (normal) or inverted?

Inverted (shift on retina is opposite)

3) What is the angular magnification of the telescope for an object at infinity? \(-5 \times\)

\[ M_{\text{ang}} = -\frac{F_r}{F_e} = -\frac{5 \text{cm}}{1 \text{cm}} = -5 \times \]

4) What is the overall focal length of the 5 and 1cm focal length lenses (i.e., not including the viewer's eye)? \( \infty \)

A focal telescope, so parallel ray is emitted parallel \( \Rightarrow f_{\text{total}} = \infty \)

5) Considering both the telescope and eye, what is the transverse (lateral) magnification for an object at infinity? \( 0 \)

A finite height image is formed for an infinitely distant (i.e., infinite height) object \( \Rightarrow \) zero magnification

6) To refocus the telescope to view an object 10m distant, without requiring the viewer's eye to change (accommodate), by how much would the distance between the two lenses need to increase or decrease?

new internal image formed @ \( \frac{1}{l_i} = \frac{1}{f} - \frac{1}{10 \text{m}} \)

\[ \Rightarrow l_i = \frac{1}{\left[ \frac{1}{5 \text{cm}} - \frac{1}{10 \text{m}} \right]} = 5.025 \text{cm} \]

\( \Rightarrow \) distance between lenses needs to increase by \( \frac{1}{q} \) mm.
Question 2: Prisms (10 points)

We found in class that for a prism with apex angle $\alpha$ the beam deflection angle $\Delta \theta \approx (n-1)\alpha$, under a specific approximation.

1. On the drawing at right, and with the prism illuminated as shown: state the approximation, define the specific angle where that approximation is applied, and briefly explain why that specific angle determines if the approximation is justified.

This equation used the paraxial approximation on Snell's law, $n \sin \theta = \text{constant} = n \theta$. It had to be valid for all angles relative to the surface normal, but for the illumination angle shown, the angle $\theta_k$ was the largest angle.

2. The table shows indexes of refraction for BK7 & F2 glasses for red and blue light. For a prism made of BK7 glass designed to deflect blue light by 10°, using the sign as defined above, what is the angular separation between red and blue light?

\[
\alpha = \frac{\Delta \theta}{\text{in radians}} = \frac{0.1745 \text{ rad}}{0.525} = 0.3324 \text{ rad} 
\]

\[
\Delta \theta_{\text{red}} = (n_{1R} - 1) \alpha = 0.52 \cdot 0.3324 = 0.1729 \text{ rad} = 9.905^\circ 
\]

\[
\Delta \theta_{\text{red}} - \Delta \theta_{\text{blue}} = -0.0952^\circ 
\]

An "achromatic" prism is made of two simple prisms of two different glasses with apex angles chosen so the net beam deflection angle is identical for the two design wavelengths.

3. Use the approximation $\Delta \theta = (n-1)\alpha$ to write two equations for the two prism apex angles for a 10° achromatic prism of BK7 and F2 [You don’t need to solve it, but the solution exists]

\[
\Delta \theta_{\text{red}} = (n_{1R} - 1) \alpha_1 - (n_{2R} - 1) \alpha_2 = 0.1745 \text{ rad} 
\]

\[
\Delta \theta_{\text{blue}} = (n_{1B} - 1) \alpha_1 - (n_{2B} - 1) \alpha_2 = 0.1745 \text{ rad} 
\]

4. Can such an achromatic prism be made with two glasses with "normal" dispersion? Explain why or why not.

Yes. Both BK7 & F2 have normal dispersion, meaning the index drops with increasing wavelength.
Achromatic prism solution (not required in midterm)

BLUE:

\[ \Delta \theta = \Delta \theta_1 + \Delta \theta_2 = (n_{B1} - 1)\alpha_1 - (n_{B2} - 1)\alpha_2 = \frac{10^\circ}{180^\circ} \cdot \pi = 0.1745 \text{ rad} \]

\[ \alpha_1 = \frac{\Delta \theta + (n_{B2} - 1)\alpha_2}{n_{B1} - 1} \quad [1] \]

RED:

\[ \Delta \theta = (n_{R1} - 1)\alpha_1 - (n_{R2} - 1)\alpha_2 \quad \text{substitute [1]} \]

\[ = \frac{(n_{R1} - 1)\Delta \theta}{n_{B1} - 1} + \frac{(n_{R1} - 1)(n_{B2} - 1)}{(n_{B1} - 1)} \alpha_2 - (n_{R2} - 1)\alpha_2 \]

\[ \Rightarrow \Delta \theta \left(1 - \frac{n_{R1}}{n_{B1} - 1}\right) = \Delta \theta \left(\frac{n_{B1} - n_{R1} + 1}{n_{B1} - 1}\right) = \Delta \theta \frac{n_{B1} - n_{R1}}{n_{B1} - 1} \]

\[ = \left[\frac{(n_{R1} - 1)(n_{B2} - 1)}{n_{B1} - 1} - (n_{R2} - 1)\right] \alpha_2 = C \alpha_2, \quad C = 0.018857 \]

And from [1]:

\[ \alpha_2 = \frac{\Delta \theta}{C} \frac{n_{B1} - n_{R1}}{n_{B1} - 1} = \frac{0.1745 \text{ rad}}{0.018857} \cdot \frac{0.005}{0.525} = 0.0881 \text{ rad} \]

\[ = 5.05^\circ \]

Check:

Prism 1

\[ \Delta \theta_{\text{blue}} = 0.231345 \quad \text{Prism 2} \]

\[ \Delta \theta = \Delta \theta_{\text{blue}} + \Delta \theta = 0.1745 \text{ rad} \]

\[ \Delta \theta_{\text{red}} = 0.229142 \quad \text{Total} \]

\[ \Delta \theta = 0.056825 \quad \text{rad} \]

\[ \Delta \theta = 0.1745 \text{ rad} \]

\[ = 25.247^\circ \]

\[ \text{same angle} \]
**Question 3:**
A pair of 20mm diameter lenses is used to form the optical relay below. The focal lengths are $F_1 = 10$ mm, and $F_2 = 15$ mm.

**Part A:** A point source radiating light uniformly in all directions is located at $x = y = z = 0$.
1) Sketch the path of the light through the relay imaging system, showing the top and bottom marginal rays (the rays that hit the edges of the lenses), and the positions of the intermediate and final image.

2) What is the $F/\#$ of the first lens?

\[
F/\# = \frac{\text{focal length}}{\text{full aperture}} = \frac{10\text{ mm}}{20\text{ mm}} = \frac{1}{2}
\]

3) What is the transverse (lateral) magnification of the overall imaging system?

This is a cascade of 2 1:1 imaging systems, each with magnification $-1 \rightarrow$ overall magnification is 1.

4) What is the numerical aperture of the light forming the final image?

N.A. of output image is $\eta_{out} \cdot \sin \Theta = \sin \Theta = \sin \tan^{-1} \left( \frac{10}{90} \right) = 0.3162$

5) What fraction of the source power is captured by the overall imaging system?

Need to know $h_0 = H \cdot \frac{F_i}{F_L}$ (by similar triangles)

\[= 10\text{ mm} \cdot \frac{10\text{ mm}}{15\text{ mm}} = 6.6\text{ mm}, \text{ and } \Theta = \tan^{-1} \left( \frac{6.6}{20} \right) = 18.43^\circ\]

Then solid angle $\frac{\Omega}{4\pi} = \frac{2\pi \left( 1 - \cos \left( 18.43^\circ \right) \right)}{4\pi} = 0.0257, 2.57\%$
6) Sketch the path of light through the system for the new source position, showing the chief ray and marginal rays.
7) What fraction of the source power is captured by the overall imaging system?

The only ray that make it through the system is the marginal ray, representing an infinitely small fraction of the source power \( \Rightarrow 0\% \) of energy is captured.

A lens which is positioned at an image plane is called a field lens.
8) Find the focal length of a field lens that will increase light collection for the off-axis point source, and
9) Sketch the path of light through the system with this lens in place.

We need to bend the light from the 1st image into the aperture of the second lens; a positive field lens is needed. If we bend the parallel ray to the top of lens 2, all the aperture will be filled. A lens with focal length 15 mm does that. Note that a lens with more power could also be used.
Question 4:

A long slab waveguide is made from a sheet of \( n = 1.6 \) glass, coated top and bottom with a cladding of index 1.575. The input face is polished at a 45° angle, and the output face polished perpendicular to the optical axis, as shown below. Light is incident on the angled face from all possible angles, and coupled without absorption to the output face.

1) What range of angles internal to the waveguide \( \theta_{\text{internal}} \), using the sign defined in the diagram above, can be directly launched from the angled input face? (this means coupled into the waveguide core, before the first internal reflection)

Maximum launch angles come from grazing incidence, where \( \theta_i = \pm 90° \) w.r.t. surface normal. Then \( \theta_e = sin^{-1}(\frac{1}{n_{\text{core}}}) = 38.68° \)

\[ \Rightarrow \theta_{\text{internal}} \text{ ranges from } 45° - \theta_e \text{ to } 45° + \theta_e, \]
\[ 6.318° \text{ to } 83.68° \]

But with \( \theta_{\text{int}} \) defined as shown,

\[ -83.68° \leq \theta_{\text{int}} \leq -6.318° \]

2) What range of angles internal to the waveguide \( \theta_{\text{internal}} \) will be guided without loss to the waveguide’s output face?

Internal critical angle \( \theta_{\text{crit}} = sin^{-1}(\frac{n_{\text{clad}}}{n_{\text{core}}}) = sin^{-1}(\frac{1.575}{1.6}) = 79.85° \)

So in terms of \( \theta_{\text{internal}} \), guided angles are

\[ -10.14° \leq \theta_{\text{int}} \leq 10.14° \]
For the long waveguide, the launched ($\theta_{\text{init}}$ negative) angles will bounce both an odd and even number of times, and be incident on the end face with positive and negative angles within the guided range.

\[ \Rightarrow \] 6.318° to 10.14° (internal), and

\[ \theta_{\text{ext}} = \sin^{-1}(1.6 \sin 6.318°) = 10.14° \]
\[ \sin^{-1}(1.6 \sin 10.14°) = 16.36° \]

\[ \Rightarrow 10.14° < |\theta_{\text{ext}}| < 16.36° \]
Question 5: Concepts

1) A glass singlet lens has at least 5 **degrees of freedom** a lens designer might define in a ray tracing program. Name them.

   - Radius of curvature of 1st & 2nd surfaces
   - Center thickness, lens diameter, and glass used.
   - Could also say 2 angles of tilt, and 2 directions of lens decenter, and distance from previous & following surfaces... but these are not attributes of the lens itself.
   - But if the lens is aspheric, it can have many higher order surface coefficients.

2) Somebody without their prescription reading glasses can read smaller text in **bright sunlight** than **dim candlelight**. Why?

   In bright light the pupil contracts, extending the depth of field for an ideal lens, and reducing aberrations for a non-ideal lens.

3) A plane wave of light in air is incident normally on an infinite surface area anti-reflection-coated block of transparent glass. Name three attributes of this light that are the same in the air as in the glass.

   - Frequency is invariant to media, always.
   - Since AR coated, intensity is unchanged.
   - Since normally incident, direction is unchanged.

4) What assumption about the electromagnetic field allowed reduction of the **Wave Equation** to the **Helmholtz Equation**?

   That the field is oscillatory with a single frequency (time dependence as $e^{j2\pi ft}$)

5) The **plane wave** and the **spherical wave** are both approximations that cannot be found in nature. For each one, why?

   - Plane waves have infinite energy, over all space.
   - Spherical waves have infinite intensity at origin.