

# ECE 181

## BEAM OPTICS

### I. OBJECTIVES

To study the properties of a Gaussian beam, as it propagates in free space and through optical components, e.g. lenses. To investigate also how to select a Gaussian beam output from a laser resonator that can support multi-mode operation.

### II. REFERENCE

B. Saleh and M. Teich, Fundamentals of Photonics, Ch. 3, pp. 81-106; Ch.9, pp. 336-337.

### III. BACKGROUND.

To use lasers intelligently in applications, it is important to understand the basic properties of laser beams. Lenses and other optical elements are also used to manipulate beam properties, e.g. beam shaping, beam expansion, etc.

The spatial characteristics of a laser beam are determined by the laser resonator. Most lasers have spherical mirror Fabry-Perot resonators that support Hermite-Gaussian spatial modes. Usually, only the lowest order transverse resonator mode oscillates, resulting in a Gaussian output beam. The location and the radius of the beam waist depend on the laser resonator parameters.

Gaussian beams are paraxial approximations to the wave equation. An expression for the complex amplitude  $U(\mathbf{r})$  of the Gaussian beam is:

$$U(\mathbf{r}) = A_0 \frac{W_0}{W(z)} \exp\left[-\frac{\rho^2}{W^2(z)}\right] \exp\left[-jkz - jk \frac{\rho^2}{2R(z)} + j\zeta(z)\right]$$

(3.1-7)

Gaussian-Beam  
Complex Amplitude

$$W(z) = W_0 \left[ 1 + \left( \frac{z}{z_0} \right)^2 \right]^{1/2}$$

$$R(z) = z \left[ 1 + \left( \frac{z_0}{z} \right)^2 \right]$$

$$\zeta(z) = \tan^{-1} \frac{z}{z_0}$$

$$W_0 = \left( \frac{\lambda z_0}{\pi} \right)^{1/2}$$

(3.1-8)

(3.1-9)

(3.1-10)

(3.1-11)

Beam Parameters

Here  $W(z)$  and  $R(z)$  are measures of the beam width and wavefront radius of curvature. The parameter  $z_0$  is known as the Rayleigh range.

**Intensity.**

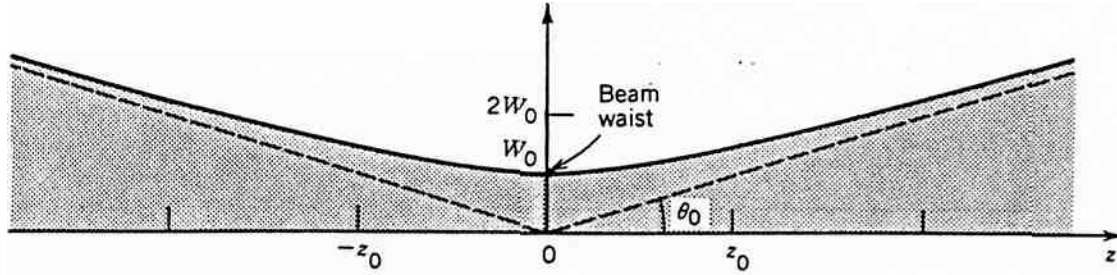
The optical intensity  $I(r) = |U(r)|^2$  is a function of the axial and radial distances  $z$  and  $\rho = (x^2 + y^2)^{1/2}$ ,

$$I(\rho, z) = I_0 \left[ \frac{W_0}{W(z)} \right]^2 \exp \left[ -\frac{2\rho^2}{W^2(z)} \right] \tag{3.1-15}$$

where  $I_0 = |A_0|^2$ . At each value of  $z$  the intensity is a Gaussian function of the radial distance  $\rho$ . This is why the wave is called a Gaussian beam. The Gaussian function has its peak at  $\rho = 0$  (on axis) and drops monotonically with increasing  $\rho$ . The width  $W(z)$  of the Gaussian distribution increases with the axial distance  $z$ .

**Beam Radius**

Within any transverse plane, the beam intensity assumes its peak value on the beam axis, and drops by the factor  $1/e^2 = 0.135$  at the radial distance  $\rho = W(z)$ . Since 86% of the power is carried within a circle of radius  $W(z)$ , we regard  $W(z)$  as the beam radius (also called the beam width). The minimum value  $W_0$  in the plane  $z = 0$ , called the beam waist. Thus  $W_0$  is the waist radius. The waist diameter  $2W_0$  is called the spot size. The beam radius increases gradually with  $z$ , reaching  $\sqrt{2} W_0$  at  $z = z_0$ , and continues increasing monotonically with  $z$  (see Fig.1):



For  $z \gg z_0$  Eq. 3.1-8 becomes:  $W(z) \approx \frac{W_0}{z_0} z = \theta_0 z$ ,

where  $\theta$  is the **beam divergence**. Using (3.1-11), we can also write:

$$\theta_0 = \frac{\lambda}{\pi W_0} \quad (3.1-19)$$

### ***Depth of Focus***

Since the beam has its minimum width at  $z = 0$ , as shown in Fig.1, it achieves its best focus at the plane  $z = 0$ . In either direction, the beam gradually grows "out of focus." The axial distance within which the beam radius lies within a factor  $\sqrt{2}$  of its minimum value (i.e., its area lies within a factor of 2 of its minimum) is known as the depth of focus. It can be seen from (3.1-8) that the depth of focus is twice the Rayleigh range:

$$2z_0 = \frac{2\pi W_0^2}{\lambda} \quad (3.1-21)$$

## **V. EXPERIMENTS.**

The radius of curvature of wavefront  $R$  and beam radius  $W$  at any point along the propagation direction  $z$  characterize a Gaussian beam completely. However, determining the curvature of wavefront requires a difficult interferometric measurement. Equivalently, we can characterize the beam by measuring  $W$  at two different locations along the propagation axis. These measurements allow us to calculate all the beam parameters.

### **A. Laser Output Intensity Profile Measurement.**

In this part of experiment you will measure the intensity profile of a He-Ne laser beam and verify that it is a Gaussian. This can be done by measuring the intensity in the plane transverse to the propagation direction using a CCD camera. You will measure the beam radius of a diverging beam at two positions along the direction of propagation and deduce all the beam parameters from these measurements.

1. Measure the beam radius at two different locations along the z-axis.
  - Place CCD camera at the distance 2m from the laser output.
  - Adjust optical attenuator and camera exposure time to avoid image saturation.
  - Capture the image and save it as bmp-file.
  - Using MatLab plot your data and try to fit a Gaussian curve to it.
  - Determine the beam radius  $W(z_1)$ .  
(To calculate the beam radius, find the location where the intensity drops down to  $1/e^2 = 0.135$  of its peak value. Remember that each pixel is  $7.5\mu\text{m}$ .)
  - Place CCD camera at the distance 3m from laser output and find the beam radius  $W(z_2)$  at that location.
2. Find  $Z_0$ ,  $W_0$ ,  $z_1$ ,  $z_2$ ,  $R(z_1)$ , and  $R(z_2)$ , from found values of  $W(z_1)$ , and  $W(z_2)$  and using Eq. 3.1-8, 3.1-9. Note that you do not know the values of  $z_1$  and  $z_2$ , but only their difference  $z_2 - z_1$ . Wavelength of the He-Ne laser is  $\lambda = 0.633\mu\text{m}$ .
3. Calculate the laser beam divergence  $\theta = \lambda/\pi W_0$ .

### **B. Gaussian Beam Transformation by a Lens.**

An ideal lens does not change the transverse profile of an optical field, so the Gaussian beam will remain a Gaussian at every point in the system. But the wavefront curvature will be changed, as the beam passes through the lens, resulting in new values of waist position and waist diameter. Since the beam waist is proportional to the focal length of lens  $F$ , and the depth of focus is proportional to  $F^2$ , the measurements are more accurate if the lens with longer focal length is used.

In this part of experiment you will measure the beam waist diameter after focusing it by lens and calculate depth of focus.

(In order to properly compute the transformation of a Gaussian beam by a lens the initial beam parameters and the lens focal length must be known.)

1. Beam expanding, collimating, and focusing.

Using the procedure described in Laboratory Technique part study how to:

- Expand and spatial filter the output laser beam (make a spherical wave).
- Collimate laser beam (make a plane wave)
- Find the focal plane and measure the focal length of an arbitrary lens (this lens will be used in the next part of experiment).

2. Measure the waist diameter of focused Gaussian beam.

- Remove the spatial filter and collimating lens used in part 1.

- Place lens with measured focal length at 2m from the laser output.
- Place CCD camera after the lens at distance closed to the focal plane and observe a beam spot on the monitor. Translate the CCD camera to find the smallest spot size.
- Measure the beam spot diameter  $2w_0$  as you done it in part A.
- Compare measured value of  $2w_0$  with calculated, using a formula:

$$2w_0 = \left( \frac{4\lambda}{\pi} \right) \left( \frac{F}{D} \right) \quad (3.2-17)$$

where D is the diameter of the beam incoming on the lens. ( $D = 2W(z_1)$  and it was measured in part A.)

- Calculate the depth of focus:

$$\text{DOF} = \left( \frac{8\lambda}{\pi} \right) \left( \frac{F}{D} \right)^2$$

DOF =  $2z_0$  and was derived from F.3.1-21 and 3.2-17.

### C. Beam Relaying by a sequence of identical lenses.

- Place three identical lenses of focal length F and separated by the distance  $d = 2F$  into the Gaussian laser beam.
- Using the CCD camera, measure the waist radius in the focal plane of the last lens.
- Compare with the waist radius  $w_0$  measured for single lens in B2.

### D. Hermite-Gaussian Beams and mode selection of laser resonator.

Laser resonators that have spherical mirrors can often support many transverse electric and magnetic modes denoted as  $TEM_{l,m}$ . Each pair of indices  $(l,m)$  defines a transverse mode with an associated special distribution. The  $(0,0)$  transverse mode is the Gaussian beam. Modes of higher  $l$  and  $m$  values form Hermite-Gaussian beams.

Most applications of gas laser require that laser operates in the lowest order  $TEM_{00}$  transverse mode because it has the smallest beam diameter and can be focused to the smallest spot size.

In this part of experiment we will demonstrate that an aperture of controllable size placed inside the resonator may be used to selectively eliminate undesired higher order modes.

## LABORATORY TECHNIQUES

### COLLIMATING A LIGHT BEAM

#### I. OBJECTIVE

To convert a diverging spherical wave into a plane wave for optical processing systems.

#### II. BACKGROUND

Collimating lenses are used for two purposes: (a) collimating a diverging light beam, and (b) focusing a collimated light beam to a point. These lenses are usually achromatic, i.e. consist of two or more elements, and are highly directional: to minimize aberrations they must be aligned with the optical axis of the system and the correct side must face the collimated beam; the side with the greatest curvature usually should face the collimated beam.

There are two basic methods for collimation: (a) using an optical flat, and (b) using a mirror and an iris. An *optical flat* is an optical component with very flat ( $\lambda/20$ ) surfaces, which are usually parallel to each other. When this flat is placed at an angle with respect to the optical axis of an incident beam, reflections from the two surfaces will interfere with each other, as shown in Fig. 1. When the beam is collimated, i.e. a plane wave, the interference pattern will have a large uniform central area since the two reflected plane waves are traveling parallel to each other.

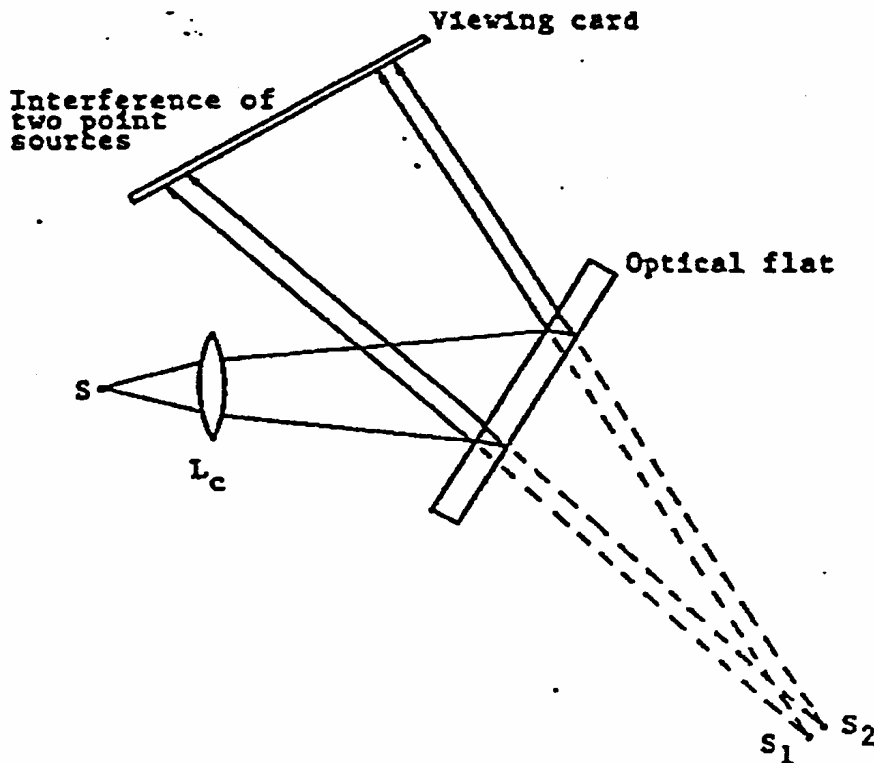


Fig. 1. Examining the degree of beam collimation with an optical flat.

Some flats have surfaces that are at a slight angle to each other, forming a *wedged flat*. In this case the interference pattern due to a collimated beam will be alternating bright and dark fringes that are spaced as far apart as possible since the two reflected plane waves will be interfering at a slight angle. This kind of optical flat lends itself to a form of "calibration": by rotating the wedged flat in a suitable mount, the fringe pattern due to a previously collimated beam can be set to be either horizontal or vertical. Thus, any incident beam can be easily collimated by adjusting the collimating lens until the appropriate set fringe pattern is achieved. The setup for collimating using a mirror and an iris is shown in Fig. 2.

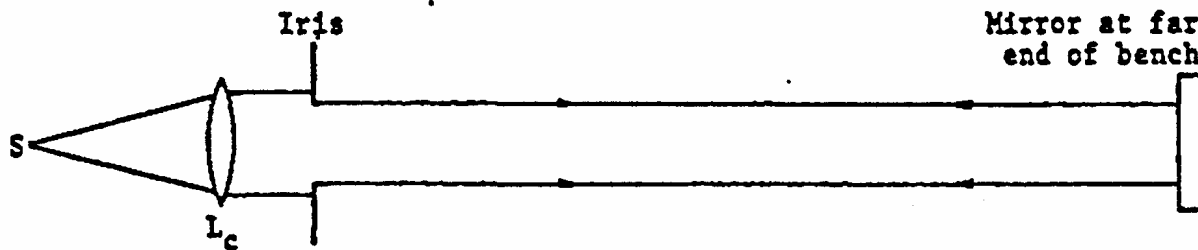


Fig. 2. Beam collimation using a mirror and an iris.

### III. PROCEDURE

#### Collimating with an optical flat:

1. Place the collimating lens approximately one focal length away from the diverging source (usually the output of a spatial filter). This will give an output beam that is almost collimated.
2. Insert the optical flat, at an angle, in the output beam path and place a white card to view the interference pattern. The angle should be chosen such that a complete interference pattern can be seen.
3. Adjust the position of the collimating lens until the desired interference pattern is obtained (uniform central area for parallel flat, largest fringe pattern for wedge flat).

#### Collimating with a mirror and an iris:

1. Place the collimating lens approximately one focal length away from the diverging source to get an almost collimated beam.
2. Insert an iris right at the output of the lens, center it on the beam, and adjust its size to allow about 80% of the light through.
3. Place an adjustable mirror at the far end of the optical bench. Adjust it until the reflected beam is centered on the iris.
4. Adjust the position of the collimating lens until the reflected beam size is the same as the forward beam through the iris.

## **LABORATORY TECHNIQUES**

### **PRECISE LOCATION OF THE FOCAL PLANE**

#### **I. OBJECTIVE**

To provide a very simple and accurate technique for locating filters at the focal plane.

#### **II. PROCEDURE**

1. Place the filter in its holder in the path of the focused beam and adjust its position for minimum light spot size.
2. Place a white card near the filter to observe the transmitted light from the filter. If the filter has a slightly irregular surface, i.e. the surface is somewhat diffuse, a speckle pattern will be seen on the card.
3. Alternatively, a piece of Scotch tape (type 810) taped tightly across a similar mount to that of the filter can be used in place of the filter if a speckle pattern is difficult to generate with the filter itself.
4. Slowly adjust the position of the filter (or the mounted tape) along the optical axis until the largest speckle pattern is seen on the card. This will indicate the best focus because the speckle size is inversely proportional to the focused spot size.
5. If the mounted tape was used to determine the focal plane, replace it with the filter. Be careful not to disturb the position of the holder.