Cross talk of wavelength-multiplexed quasi-infinite holograms

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The cross talk of wavelength-multiplexed quasi-infinite (written in polychromatic light) holograms is evaluated. Under plausible conditions this cross talk is negligible. The typical information capacity limit of such holograms is evaluated as 10^{12} bits for a 1-cm^3 volume. © 1998 Optical Society of America

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In view of the fast developments in semiconductor frequency-tuned lasers, wavelength multiplexing of volume holograms is considered promising for optical memory systems.[1–3] However, with wavelength multiplexing as well as with other multiplexing methods, there exists cross talk that limits memory capacity.[1,4–7] This cross-talk noise is associated with the finite longitudinal size of the recording material.[5] A method that allows one to suppress the cross talk for wavelength-multiplexed storage and uses holograms that are limited in the longitudinal direction, called quasi-infinite holograms, was presented in Ref. 8. Quasi-infinite holograms can be recorded by use of two short light pulses, two stochastic fields, or averaged interference of two monochromatic waves whose wavelengths are swept during the recording process. In this Letter we investigate the cross-talk noise of wavelength-multiplexed quasi-infinite holograms and estimate the information capacity limit of such holographic storage systems.

Consider, as an example, a volume hologram recorded with two counterpropagating short pulses, as illustrated in Fig. 1. The spatial interference pattern along the longitudinal direction is weighted by the mutual coherence function of these partially coherent fields.[9] Let $g(t)\exp(i\omega_c t)$ be the mutual coherence function of the signal and the reference waves, where $g(t)$ is the envelope and ω_c is the center frequency. Volume holographic materials are sensitive to spatial intensity modulation, and thus the mutual coherence function is recorded as a volume grating with amplitude varying along the longitudinal coordinate $z = vt/2$, where v is the light velocity in the medium and the factor of 2 accounts for the counterpropagation recording. This grating is described by $h(z) = g(2z/v)\exp(i\omega_c 2z/v)$. When interfering waves possess time-limited mutual coherence functions, the recorded grating will be of limited extent along the z axis and therefore can be recorded completely in a material of finite longitudinal dimension.

For the hologram reconstruction process, the quasi-infinite hologram can be seen as a superposition of monochromatic gratings of infinite spatial extent in z, with the grating amplitude weighted by the Fourier transform of $h(z)$. In such a description, when a quasi-infinite hologram is reconstructed by a monochromatic plane wave of frequency ω, only the Bragg-matched, infinite gratings will determine the amplitude diffraction efficiency. Consequently, the amplitude diffraction efficiency's dependence on frequency ω can be expressed by the Fourier transform of the mutual coherence function; i.e., the normalized diffraction efficiency $G(\xi)$ will be proportional to the mutual spectral density[9]:

$$G(\xi) = \int_{-\infty}^{\infty} g(t)\exp(-i\xi t)\mathrm{d}t\,, \tag{1}$$

where $\xi = \omega - \omega_c$. Furthermore, notice that even for recording of wide angular bandwidth signals, the quasi-infinite holograms will not be truncated by the material boundaries, and Eq. (1) will be valid for all the components of the angular bandwidth. In contrast, recording such holograms with monochromatic light will result in gratings that are truncated by the material boundaries, introducing windows that vary with the angular components, which in turn leads to the variation of diffraction efficiency spectrum within the angular bandwidth.

The wavelength multiplexing of quasi-infinite holograms can be realized by use of different center frequencies $\omega_{cj} = \omega_0 + j\Delta$, where ω_0 is a center of the multiplexing frequency band, $j = 0, \pm 1, \ldots \pm J/2$ is the integer corresponding to a given information page, J is an even integer such that the total number of holograms is $(J + 1)$, and Δ is the frequency increment. The jth information page is reconstructed with monochromatic beams with the corresponding frequency $\omega_{cj} = \omega_0 + j\Delta$. The multiplexing is performed by means of recording many quasi-infinite holograms in the same volume, causing cross-talk noise during reconstruction. This cross talk can be diminished by a proper choice of mutual coherence function. For example, the mutual coherence function, $g_q(t)$, can be generated from rectangular functions

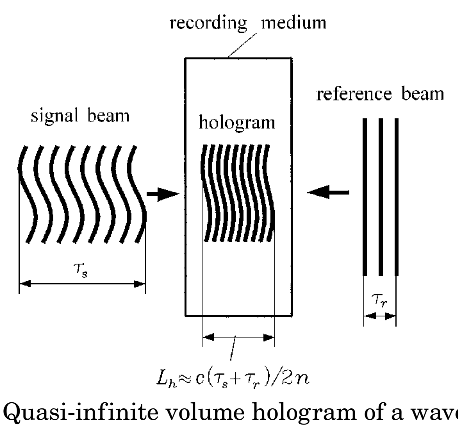

Fig. 1. Quasi-infinite volume hologram of a wave packet.

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$p(t) = (1/2T)\mathrm{rect}(t/2T)$ with unit area and parameter $T = \pi/\Delta$[8]:

$$g_q(t) = p(t) * \cdots * p(t), \qquad (2)$$

where $*$ denotes convolution operation applied $q - 1$ times and the function $g_q(t)$ has the boundaries $-qT$ and qT. The length of the interference pattern, $L_c = qT/v$, should be less than the material length L_h. The amplitude diffraction efficiency for the mutual coherence function of Eq. (2) follows from Eq. (1):

$$G_q(\xi) = (\sin \xi T/\xi T)^q, \qquad (3)$$

where $\xi = \omega - \omega_{cj} = \omega - (\omega_0 + j\Delta)$. This function has its maximum at $\xi = 0$ and zeros at $\xi = n\Delta$ for any integer $n \neq 0$. For recall of the jth information page, the reconstruction wave is tuned to the jth central frequency $\omega_{cj} = \omega_0 + j\Delta$, and only the jth hologram is reconstructed without any cross talk.

In practice, since the frequency bandwidth of optical radiation used for hologram recording is always limited, the mutual coherence function spreads to infinity, thus violating the condition that a quasi-infinite hologram must be obtained and leading to a certain level of cross talk. To account for such physical limitations we chose to truncate the mutual spectral density of the reference and signal radiation $G_q(\xi)$ (which is proportional to the diffraction efficiency), using a square window $\mathrm{rect}(\xi/2\Omega)$. The introduced bandwidth 2Ω limits the number of multiplexed holograms to the value $J \approx 2\Omega T/\pi$. We also notice that since during the changing of the multiplexing frequency ω_{cj} the mutual spectral density of jth hologram is shifted by the value $j\Delta$, the total spectral bandwidth employed is 4Ω.

A new mutual spectral density $Gq(\xi)\mathrm{rect}(\xi/2\Omega)$ can be generated in practice, since it possesses the same zero crossings as $G_q(\xi)$ and cross-talk noise will not be generated. However, its corresponding mutual coherence function,

$$g_q{}'(t) = \frac{1}{2\pi}\int_{-\Omega}^{\Omega} G_q(\xi)\exp(i\xi t)\mathrm{d}\xi = g_q(t) * \sin(\Omega t)/\pi t\,, \qquad (4)$$

will need to be recorded in a material of infinite longitudinal extent (for $q = 1$; see Fig. 2). The boundaries of the ideal mutual coherence function, $-qT$ and qT have been extended to infinity owing to the convolution with the $\sin(\Omega t)/(\pi t)$ [see Eq. (2)]. The material boundaries introduce truncation of the mutual coherence function of Eq. (3) by a rectangular window $\mathrm{rect}(t/2T_h)$, where $T_h = L_h/v$. Finally, the diffraction efficiency of such a truncated hologram is

$$G_q{}'(\xi) = \int_{-T_h}^{T_h} g_q{}'(t)\exp(-i\xi t)\mathrm{d}t = G_q(\xi)\mathrm{rect}(\xi/2\Omega) * 2\sin(T_h\xi)/\xi\,. \qquad (5)$$

Owing to the convolution operation, we can observe from Eq. (5) that the spectrum $G_q{}'(\xi)$ is different from the spectrum $G_q(\xi)\mathrm{rect}(\xi/\Omega)$, causing cross talk.

We define a cross-talk factor $Q(\xi)$, which is the deviation of the practically achievable diffraction efficiency $G_q{}'(\xi)$ from that which does not introduce cross talk, $G_q(\xi)\mathrm{rect}(\xi/2\Omega)$:

$$Q(\xi) = G_q{}'(\xi) - G_q(\xi)\mathrm{rect}(\xi/2\Omega) = -\left(\int_{-\infty}^{-T_h} + \int_{T_h}^{\infty}\right) g_q{}'(t)\exp(-i\xi t)\mathrm{d}t\,. \qquad (6)$$

We next estimate the integral given by Eq. (6), using an asymptotic evaluation of $g_q{}'(t)$ from Eq. (4) for a value of t that is a large parameter following the boundary value theorem of Fourier integral evaluation.[10] The principal term of the asymptotic evaluation of the Fourier integral Eq. (4) is given by the boundary values of the function $G_q(\xi)$, if these values are not zero, yielding

$$g_q{}'(t) \sim G_q(\Omega)\sin(\Omega t)/\pi t\,. \qquad (7)$$

Substituting Eq. (7) into Eq. (6) yields

$$Q_q(\xi) \sim \frac{1}{\pi} G_q(\Omega)\{\mathrm{si}[(\Omega + \xi)T_h] + \mathrm{si}[(\Omega - \xi)T_h]\},$$

where the integral sine is given by $\mathrm{si}(x) = -\int_x^{\infty}(\sin u/u)\mathrm{d}u$. Notice that the $\mathrm{si}(x)$ function in Eq. (7) decreases very rapidly owing to the large values of T_h (i.e., the longitudinal extent of the recording material is large compared with the support of the mutual coherence function). For cross-talk noise calculation we need values of $|Q_q(\xi)|^2$:

$$|Q_q(\xi)|^2 \sim \frac{1}{\pi^2} G_q{}^2(\Omega)\{\mathrm{si}^2[(\Omega + \xi)T_h] + \mathrm{si}^2[(\Omega - \xi)T_h]\}. \qquad (8)$$

For characterization of the cross-talk noise we next consider reconstruction of the hologram corresponding to $j = 0$, i.e., reconstruction with center frequency ω_0. The normalized diffraction efficiency for this hologram $G_q{}'(0) \approx 1$, and the cross-talk diffraction efficiencies $G_q{}'(-j\Delta)$ are generated. The amplitude of the jth cross-talk noise term is $N_j = S_j Q(-j\Delta)$, where S_j is the amplitude of the jth signal with $j \neq 0$. The total amplitude noise N that is due to cross talk is a coherent sum,

$$N = \left(\sum_{j=-J/2}^{-1} + \sum_{j=1}^{J/2}\right) S_j Q(j\Delta),$$

where $J + 1 \approx J$ is the total number of multiplexed holograms. The power of the cross-talk noise is

$$\langle |N|^2 \rangle = \left(\sum_{j=-J/2}^{-1} + \sum_{j=1}^{J/2}\right) |Q(j\Delta)|^2, \qquad (9)$$

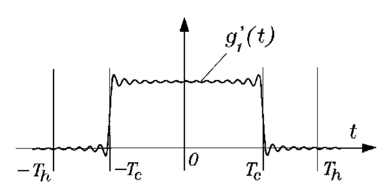

Fig. 2. Recording of the nonideal coherence function in a medium with a finite longitudinal dimension.

where we assume that S_j are statistically independent complex random quantities with average power $\langle |S_j|^2\rangle = 1$ and use the fact that the number of holograms is large, such that the sum of the cross products of the type $S_k Q(k\Delta) S_l^* Q^*(l\Delta)$ is close to zero.

We estimate the value $\langle |N|^2\rangle$ by use of Eq. (8). The spectral boundaries of our recorded mutual spectral densities satisfy $J\Delta \le 2\Omega \le (J+1)\Delta$. We need to find Ω such that the function ${G_q}^2(\Omega)$ makes a maximum contribution to the cross-talk noise for a given value of J. This occurs when $\Omega = (J/2 + 1/2)\Delta$, providing the value ${G_q}^2(\Omega) \approx 1/(\Omega T)^{2q}$. At the multiplexing frequencies closest to the Ω boundaries [i.e., $\xi = \pm(J/2)\Delta$], the argument of $\mathrm{si}^2[(\Omega \pm \xi)T_h] \equiv \mathrm{si}^2(x)$ in Eq. (8) is $x_{J/2} = (\Delta/2)T_h$, which is also a minimum value of x for all multiplexing frequencies ξ_j. Again using the fact that T_h is chosen to be a large parameter, we employ asymptotic approximation $\mathrm{si}^2(x) \sim 1/x^2$. Using this approximation and the choice and definition of Ω, we can rewrite Eq. (8) as

$$|Q_q(j\Delta)|^2 \sim \frac{1}{\pi^2}\, 1/(\Omega T)^{2q} \left\{ \left[\left(\frac{J+1}{2}\,\Delta + j\Delta \right) T_h \right]^{-2} + \left[\left(\frac{J+1}{2} \right)\Delta - j\Delta T_h \right]^{-2} \right\}.$$

Substituting the above relation into Eq. (9) yields

$$\langle |N|^2\rangle \sim \frac{2}{\pi^2(\Omega T)^{2q}} \sum_{k=1}^{J/2} \frac{4}{(2k+1)^2 (T_h\Delta)^2}, \qquad (10a)$$

where we have introduced the summation over index k, defined by $k = J/2 - j$, and a factor of 2 was introduced to account for the summation over the negative indices in Eq. (9). For large values of J, the series in relation (9) converges to

$$\langle |N|^2\rangle \sim \frac{1}{\pi^2(\Omega T)^{2q}} \left(\frac{T}{T_h} \right)^2. \qquad (10b)$$

Relation (10b) shows that for a large number of multiplexed holograms, $\Omega T \approx \pi J/2$ is large, significantly decreasing the multiplexing cross talk. To satisfy the condition $T_h \gg qT$, we can choose the hologram length to be twice the spatial extent of the mutual coherence function, i.e., $T_h \approx 2qT$, yielding from relation (10b)

$$\mathrm{SNR} \sim (2\pi q)^2 (\pi J/2)^{2q}. \qquad (11)$$

For example, if $J = 10^3$, and $q = 2$, the resultant signal/noise ratio is $\sim 10^{14}$, indicating that the cross talk associated with the finite-frequency bandwidth is negligibly small for quasi-infinite holograms.

Finally, we estimate the information capacity limit I of wavelength-multiplexed quasi-infinite holograms, applying the Shannon formula[11]:

$$I = MJ \log_2 \mathrm{SNR}. \qquad (12)$$

Here M is the number of pixels in an information page, J is the number of wavelength-multiplexed holograms, and the signal/noise ratio is provided by relation (11). Consider the dependence of information capacity on the geometrical parameters of the recording volume. It can be seen that $J \approx \zeta(L_h/\lambda)(1/2q)$, where $\zeta = 4\Omega/\omega_0$ is the ratio of the whole frequency interval 4Ω used for multiplexing to the center frequency ω_0; λ is the average wavelength of radiation inside the recording medium; and $M = SA^2/\lambda^2$, where S is the cross-section area and A is the angular aperture of the signal beam inside the hologram. The volume of the recording material used for multiplexing quasi-infinite holograms is $V_h \approx L_h S$. Taking this into account, we obtain $MJ = (A^2\zeta V_h)/(2q\lambda^3)$. Correspondingly, the signal/noise ratio is

$$\mathrm{SNR} \approx (2\pi q)^2 \left(\frac{\pi\zeta}{4q}\,\frac{L_h}{\lambda} \right)^{2q},$$

and the information capacity is

$$I \approx \frac{A^2\zeta}{q}\,\frac{V_h}{\lambda^3} \log_2 \left[2\pi q \left(\frac{\pi\zeta}{4q}\,\frac{L_h}{\lambda} \right)^q \right].$$

For example, in the case of a cube-shaped recording material with 1-cm sides, $q = 2$, $\lambda = 0.5$ μm, and $A = \zeta = 0.2$, we obtain $I \sim 10^{12}$ bits.

In summary, the cross talk of quasi-infinite holograms associated with the finite-frequency bandwidth is shown to be extremely small, which allows one to consider the method of quasi-infinite holograms as a means of developing virtually orthogonal multiplexing of two-dimensional spatial signals.

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