Negative radiation pressure on gain medium structures

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We demonstrate negative radiation pressure on gain medium structures, such that light amplification may cause a nanoscale body to be pulled toward a light source. Optically large gain medium structures, such as slabs and spheres, as well as deep subwavelength bodies, may experience this phenomenon. The threshold gain for radiation pressure reversal is obtained analytically for Rayleigh spheres, thin cylinders, and thin slabs. This threshold vanishes when the gain medium structure is surrounded by a medium with a matched refractive index, thus eliminating the positive scattering forces.

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In the past four decades, since the pioneering work of Ashkin [1], laser-induced optical forces on neutral bodies have been a subject of great interest. Research on this topic has included the trapping and manipulation of small particles [2], cavity optomechanics [3], and recently, optical forces on nanoscale waveguides [4–8]. The vast majority of these studies has been focused on structures made of passive dielectric materials, whereas forces on gain media have received little attention. Gain media may be modeled by a complex permittivity with an active imaginary part; the latter may be responsible for the reversal of mechanical effects, such as the acceleration instead of deceleration of charged particles traversing through a gain medium [9,10]. The gain medium itself, however, may be subject to forces that are due to the stimulated emission of radiation. In this regard, it has been shown that the direction of the torque on an absorptive body exerted by an incident wave with a helical phase is reversed when the body has gain instead [6]. It has also been demonstrated that the resonant peaks of radiation pressure on active Mie spheres may be inverted to dips [11]. Moreover, negative radiation pressure on cold atoms due to their optical gain has been observed [12].

In this Letter, we demonstrate that gain medium structures may be subject to negative radiation pressure, such that an object tends to be pulled toward the source of the incident wave. Incorporation of gain media in nanoscale optical-force-based systems will therefore allow more degrees of freedom and result in novel effects and functionalities. The demonstrated effects occur in structures comparable to or larger than the material wavelength, such as gain medium slabs and Mie spheres, as well as for deep subwavelength structures, such as thin slabs and Rayleigh spheres and cylinders. In most cases, a threshold gain for radiation pressure reversal exists, and analytic expressions for it are obtained in several cases.

Let us consider a slab of relative permittivity $\varepsilon_g = \varepsilon'_g + j\varepsilon''_g$, so that under the assumption of time dependence of the form $\exp(j\omega t)$, $\varepsilon''_g < 0$ corresponds to absorption, whereas $\varepsilon''_g > 0$ corresponds to material gain. The time-averaged radiation pressure on the slab resulting from a plane wave incident perpendicularly with power density S_{inc} is given by [13]

$$F/F_0 = 1 + R - T, \tag{1}$$

where $F_0 \equiv S_{\mathrm{inc}}/c$ is the radiation pressure that would result from total absorption of the incident power; R and T are the power reflection and transmission coefficients, respectively, which may be larger than 1 if power is generated within the slab. As an example, the real part of the permittivity is chosen to be $\varepsilon'_g = 3.54^2$, corresponding to semiconductor gain media. The normalized pressure F/F_0 is plotted in Fig. 1(a) as a function of the normalized slab width d/λ_g, where $\lambda_g \equiv \lambda_0/\sqrt{\varepsilon'_g}$ is the wavelength inside the slab, in the vicinity of the first Fabry–Perot resonance, for three different values of ε''_g. When the slab is lossless and passive ($\varepsilon''_g = 0$), then at the Fabry–Perot resonance points, the transmission and reflection coefficients are $T = 1$ and $R = 0$, respectively, and consequently the force vanishes (dashed-dotted curve). For an absorptive $\varepsilon''_g = -0.05$, the force becomes positive at its minimal value due to the momentum transferred to the slab (dashed curve). However, once gain is introduced into the slab ($\varepsilon''_g = 0.05$), the transmission coefficient is larger than 1, and the force reverses its sign (solid curve). Thus, the negative radiation pressure is due to the recoil generated by the amplified radiation.

Increasing the slab width allows for higher amplification of the incident beam, consequently increasing the negative radiation pressure peaks. This is seen in Fig. 1(b),

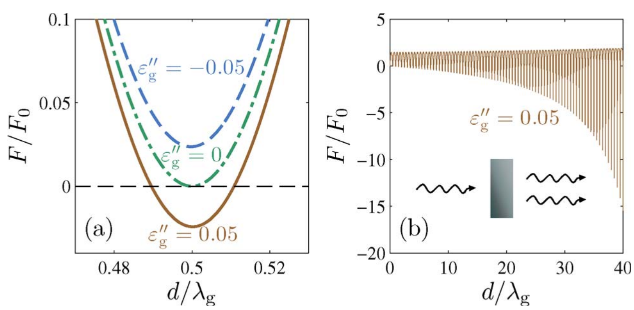

Fig. 1. (Color online) Normalized radiation pressure on a gain medium slab with $\varepsilon'_g = 3.54^2$ as a function of the normalized slab width. (a) First resonance for three values of ε''_g. (b) Several resonant peaks showing increase of negative radiation pressure with the gain medium slab width. The inset illustrates the configuration.

where the peaks reach a maximum of $F/F_0 \simeq -16$ in the shown range. The enhancement of the resulting force compared to F_0 suggests that a laser beam may be used to actuate a mechanical system that is based on a semiconductor slab. This process is limited by the noise in the gain medium, its saturation, and by the lasing threshold, reached in the considered example at $d/\lambda_g \simeq 46$. We further point out that if the slab is coated with a reflective material on one of its sides, then either amplified spontaneous emission or lasing would exert a force on the slab, without the presence of an external incident wave. The force would then be opposite to the direction of the emitted radiation, resulting in a "laser rocket" effect.

A more complex scatterer is a Mie sphere, for which there may be sharp peaks of the radiation pressure as a function of the size parameter $k_0 a$, where $k_0 = 2\pi/\lambda_0$ and a is the radius [14]. For a gain medium Mie sphere, it has been shown that the radiation pressure peaks may be inverted to dips that are still positive [11]. Negative radiation pressure on Mie spheres, however, has not been reported yet, to the best of our knowledge. To achieve a negative force, one has to suppress the backward scattering relative to the amplification of the incident wave. This may be done by either increasing the gain or by decreasing the index contrast between the sphere and the surrounding medium. At the same time, the gain must be kept lower than the threshold gain for lasing of nearby resonances, to prevent self-oscillations.

We consider a sphere having $\varepsilon_g' = 1.45^2$ immersed in a lossless surrounding medium with permittivity of $\varepsilon_m = 1.33^2$, corresponding to water. In Fig. 2(a) we plot the positive radiation force for a lossless sphere ($\varepsilon_g'' = 0$) due to an incident plane wave as a function of the size parameter in the range $k_0 a = 42$ to 46. The force is computed using the Mie scattering coefficients [15], and the results are confirmed by integrating over the Maxwell stress-tensor inside the surrounding medium [16]. We normalize the force by $F_0 \equiv \pi a^2 S_{\mathrm{inc}}/c$, where S_{inc} is the power flow density of the incident wave. The observed fluctuations correspond to the resonances of the sphere.

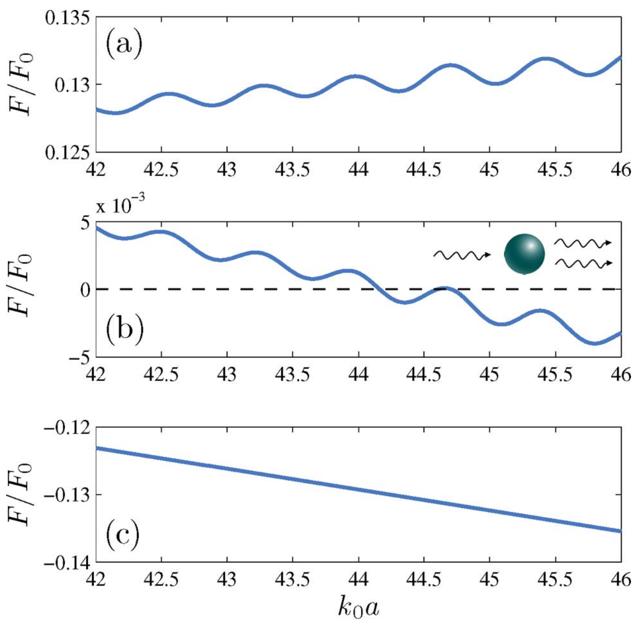

Fig. 2. (Color online) Normalized radiation force on a Mie sphere with $\varepsilon_g' = 1.45^2$ as a function of the size parameter. (a) $\varepsilon_m = 1.33^2$ and $\varepsilon_g'' = 0$. (b) $\varepsilon_m = 1.33^2$ and $\varepsilon_g'' = 2.1 \times 10^{-3}$. The inset illustrates the configuration. (c) $\varepsilon_m = 1.45^2$ and $\varepsilon_g'' = 2.1 \times 10^{-3}$.

When the sphere has an active imaginary part of the permittivity, the larger the sphere, the more amplification of light may overcome the scattering forward force to produce a negative force, as shown in Fig. 2(b) for $\varepsilon_g'' = 2.1 \times 10^{-3}$. The negative force may be further enhanced by matching the real part of the permittivity of the surrounding medium to that of the sphere, thereby virtually eliminating the scattering, and reducing the threshold gain for radiation pressure reversal to zero. In this case the negative radiation force, depicted in Fig. 2(c), becomes more pronounced and comparable to the positive force obtained with no gain.

Deep subwavelength structures may be subject to negative radiation pressure as well. We next consider a Rayleigh sphere made of gain medium, and we assume that a plane wave with electric field amplitude E_0 is incident upon it. The electrostatic polarizability of the sphere of radius a in a surrounding medium of permittivity ε_m is given by the Clausius–Mossoti relation [16], which reads

$$\alpha_0 = 4\pi\varepsilon_0\varepsilon_m a^3(\varepsilon_r - 1)/(\varepsilon_r + 2), \tag{2}$$

where $\varepsilon_r \equiv \varepsilon_g/\varepsilon_m \equiv \varepsilon_r' + j\varepsilon_r''$. The polarizability including radiation reaction is given by [17]

$$\alpha = \alpha_0 \Big/ \left(1 + j\frac{2}{3}\frac{k^3\alpha_0}{4\pi\varepsilon_0\varepsilon_m}\right), \tag{3}$$

where $k \equiv k_0\sqrt{\varepsilon_m}$. The force on the sphere reads [17]

$$F = -\frac{1}{2}kE_0^2 \mathrm{Im}(\alpha). \tag{4}$$

To obtain a negative force, the gain must be high enough to overcome the positive scattering force. The threshold gain for radiation pressure reversal is found by imposing $F = 0$, which results in a quadratic equation in ε_r'':

$$\xi\varepsilon_r''^2 - 3\varepsilon_r'' + \xi(\varepsilon_r' - 1)^2 = 0, \tag{5}$$

where $\xi \equiv \frac{2}{3}(ka)^3$. The solution is given by

$$\varepsilon_{r,th}'' = \left(3 - \sqrt{9 - 4\xi^2(\varepsilon_r' - 1)^2}\right)/(2\xi), \tag{6}$$

and the threshold gain is $\varepsilon_{g,th}'' = \varepsilon_m\varepsilon_{r,th}''$. The second solution of Eq. (5) typically represents an extremely high value of gain where the dipole approximation is invalid. For sufficiently small ka and ε_r'', we approximate

$$F \simeq -2\pi\varepsilon_0\varepsilon_m k a^3 E_0^2\left[\frac{3\varepsilon_r''}{(\varepsilon_r' + 2)^2} - \frac{2}{3}(ka)^3\left(\frac{\varepsilon_r' - 1}{\varepsilon_r' + 2}\right)^2\right]. \tag{7}$$

A negative force is obtained when the first term, which is due to the generation of power in the sphere, is larger than the second term, which is due to the scattering. The relative threshold gain is thus approximated by

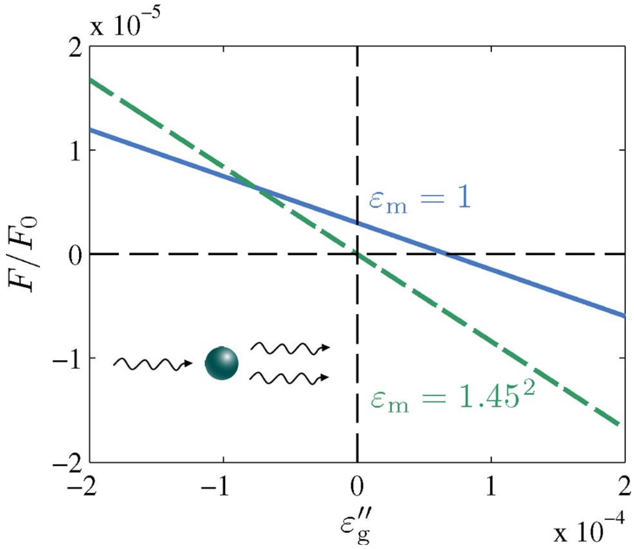

Fig. 3. (Color online) Normalized radiation force on a sphere with radius $a = 0.01\lambda_0$ and $\varepsilon'_g = 1.45^2$ as a function of the gain. The inset illustrates the configuration.

$$\varepsilon''_{\mathrm{r,th}} \simeq \frac{2}{9}(ka)^3(\varepsilon'_r - 1)^2. \quad (8)$$

One may obtain the same result by approximating Eq. (6) using $\frac{4}{9}\xi^2(\varepsilon'_r - 1)^2 \ll 1$. The threshold gain vanishes when the real part of the permittivity is matched to that of the surrounding medium ($\varepsilon'_r = 1$). We note that the same threshold gain applies for particles pushed by the evanescent tail of a waveguide mode [17].

To illustrate the threshold gain for negative radiation pressure on Rayleigh spheres, we depict in Fig. 3 the normalized force using Eq. (4) as a function of the gain ε''_g for a sphere with radius $a = 0.01\lambda_0$ and permittivity $\varepsilon'_g = 1.45^2$. We find excellent agreement of these curves with Eq. (7), where the linear behavior is evident. When the surrounding medium has $\varepsilon_m = 1$ (solid curve), the threshold gain is $\varepsilon''_g \simeq 6.7 \times 10^{-5}$. Within the range of ε''_g shown, it is seen that the negative radiation force may become stronger than the forward radiation force obtained at $\varepsilon''_g = 0$. The dashed curve corresponds to $\varepsilon_m = 1.45^2$, where the threshold gain vanishes and the negative radiation force is enhanced.

The relative threshold gain for thin cylinders or nanowires may be obtained in a similar manner to Rayleigh spheres. Considering polarization of the incident wave parallel to the cylinder axis yields

$$\varepsilon''_{\mathrm{r,th}} = \left(1 - \sqrt{1 - 4\xi^2(\varepsilon'_r - 1)^2}\right)/(2\xi), \quad (9)$$

where $\xi \equiv \frac{\pi}{4}(ka)^2$ and a is the cylinder radius. We find that Eq. (9) is valid for an optically thin slab as well, with $\xi \equiv \frac{1}{2}kd$. The approximate threshold gains when $4\xi^2(\varepsilon'_r - 1)^2 \ll 1$ are $\varepsilon''_{\mathrm{r,th}} \simeq \frac{\pi}{4}(ka)^2(\varepsilon'_r - 1)^2$ and $\varepsilon''_{\mathrm{r,th}} \simeq \frac{1}{2}kd(\varepsilon'_r - 1)^2$ for the cylinder and for the slab, respectively. Evidently, the strongest decay of the threshold gain with the dimension is for the sphere case, followed by the cylinder case, and then the slab case.

In conclusion, we have demonstrated negative radiation pressure on gain medium structures including optically thick and thin slabs, Mie spheres, Rayleigh spheres, and thin cylinders. The threshold gain for radiation pressure reversal has been shown to vanish for resonant slabs or when the surrounding medium is refractive index matched to the body. We note that if optical pumping is used, its radiation pressure can be avoided by pumping in an orthogonal axis to that of the desired motion, and symmetrically. This study suggests novel ways of actuating nanoscale optomechanical systems, thereby enriching the scope of the mechanical effects of light.

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