Self-alignment and instability of waveguides induced by optical forces

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We introduce a fundamental property of waveguides induced by the forces of the guided light, namely, the ability to self-align or be in instability. A nanoscale waveguide broken by an offset and a gap may tend to self-align to form a continuous waveguide. Conversely, depending on the geometry and light polarization, the two parts of the waveguide may be deflected away from each other, thus, being in an unstable state. These effects are unique as they rely on the presence of both the guided mode and the scattered light. Strong self-alignment forces, in both the transverse and longitudinal directions, may be facilitated by near field interaction with polarization surface charges.

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Laser light has significant mechanical effects on microscopic objects, as was initially pointed out about four decades ago [1]. In addition to the vast work on trapping and manipulation of small particles [2], much effort has been directed at cavity-based optomechanical devices, where optical forces may be considerably enhanced [3–5]. A less explored option, however, is optical forces on waveguides, which become observable at the microscopic scale [6,7].

Research of optical forces on waveguides is motivated by the growing capabilities of nanofabrication that enable new possibilities of nanoscale light manipulation [8–10]. The theory of guided light is thus being extended to include the laws of the mechanical effects of light on the guiding structure itself [11–16]. These new physical mechanisms relate the properties of the guided modes to the forces created by them. For instance, light guided between two waveguides or mirrors creates a repulsive force for an antisymmetric transverse field and an attractive force for a symmetric transverse field [12,13]. Moreover, a superposition of a symmetric and antisymmetric modes may hold the waveguide in a stable equilibrium [13,15,17]. Such phenomena can be experimentally observed in nanomechanical devices fabricated on a chip, as was recently demonstrated with a suspended silicon waveguide [18].

In this Rapid Communication, we introduce a fundamental property of waveguides, namely, the ability of a waveguide to self-align by light forces when it is perturbed by a small offset misalignment. We show that the size of the waveguide and the type of eigenmode determine whether the two misaligned parts will tend to self-align or deflect away from each other. This phenomenon is unique as it relies on both the guided waveguide eigenmode and the scattered radiation from the perturbation. A strong self-alignment force is created due to polarization surface charges that dominate the optical force with a near field interaction. We further investigate the exerted forces when a gap is introduced between the two parts of the waveguide.

The geometry under consideration is shown in Fig. 1. A single mode slab waveguide of half-width Δ and permittivity εr is broken by an offset in the x axis Δ and a gap in the z axis g. An eigenmode is incident from the left (input waveguide) carrying power Pin, most of which is transmitted to the output waveguide (Porl), while the remainder is either scattered (Pscat) or reflected back into the input waveguide. No variations in the geometry are assumed along the y axis, and, therefore, all quantities given are per unit length.

First, we examine the case of no gap g=0 and a small offset Δ. For the calculation of the fields, we take an approach of mode matching approximation, similar to that described by Marcuse [19,20]. We begin by considering a transverse electric (TE) incident mode, for which the nonzero field components are Eγ, Hx, and Hz. The even guided mode incident from the left is given inside the dielectric slab by Ei=ATE cos(kxz)exp(−jβgz), where ATE=√2μ0σ0Pinc/βg(d+γ−1), kx is the transverse wave number in the dielectric, βg is the longitudinal wave number, γ is the transverse decay constant outside the slab, and the time dependence is of the form e+jωt. The transverse electric field in each region may be represented as a sum of the guided modes and the continuous spectrum of the radiation modes (Ref. [20], pp. 314–319), corresponding to the scattered light. Explicitly, assuming the interface plane is at z=0, then for z<0 (input waveguide) this field is given by

\[ E_i = E_i + a_i E_i + \int_0^\infty dpq_{c1}(p)E_{c1} + \int_0^\infty dpq_{c1}(p)E_{c1}, \]

whereas for z>0 (output waveguide) it reads as

![FIG. 1. (Color online) A single mode slab waveguide broken by an offset and a gap.](image-url)
$$E_2 = a_2 E_1 + \int_0^\infty dpq_2(\rho) E_{22} + \int_0^\infty dpq_2(\rho) E_{02}. \quad (2)$$

In the above two equations, $a_i$ and $a_1$ are the reflection and transmission coefficients of the guided mode, respectively; $E_i$ and $E_t$ are the reflected and transmitted guided modes, respectively; $q_{12}$ and $q_{012}$ are the amplitudes of the even and odd radiation modes, respectively; $\rho = \sqrt{k_0^2 - \beta_i^2}$ is the transverse wave number of the radiation modes outside the slabs, where $\beta_i$ is their longitudinal wave number and $k_0 = \omega/c$; $E_{12}$ and $E_{012}$ are the even and odd radiation modes, respectively. The amplitudes of the guided and the radiation modes may be approximated analytically by expressions containing the overlap integral of type of Eq. (9.5-10) in Ref. [20] between the respective mode and the incident guided mode.

Generally, force densities on dielectrics may be viewed as resulting from two processes involving the polarization density $\hat{P} = \varepsilon_0 (\varepsilon_i - 1) \varepsilon \hat{E}$ [13,21–23]: (i) time-averaged interaction of effective polarization volume current densities with the magnetic field $\frac{1}{2} \text{Re} (j_0 \hat{P} \times \mu_0 \hat{H})$ and (ii) the interaction of polarization surface current densities with the electric field $\frac{1}{2} \text{Re} (j \hat{n} \cdot \hat{P}) \hat{E}$, where $\hat{n}$ is the normal to the surface of the dielectric. In the TE case, no surface charge densities are formed, and, thus, the volume force density in the $x$ direction is integrated and obtained in terms of integration over only the top and bottom surfaces

$$F_{x2} = \frac{1}{4} \varepsilon_0 (\varepsilon_i - 1) \int_0^\infty dz z [E_2^2]_{z=d+\Delta} - [E_2^2]_{z=-d+\Delta}. \quad (3)$$

Once an offset ($\Delta \neq 0$) is introduced, symmetry is broken, and scattering occurs from the discontinuity giving rise to a transverse force.

For the evaluation of the force on the output waveguide, the field expression of Eq. (2) is substituted into Eq. (3). At this point, we are interested in small offsets, so that only terms up to the first order of $\Delta$ are kept. Noting that cross-products of the even radiation modes with the odd radiation modes are of order larger than $\Delta$, and using the symmetry properties of the modes, the expression for the force reads as

$$F_{x2} = \varepsilon_0 (\varepsilon_i - 1) \text{Re} \int_0^\infty dz E_2^2 \int_0^\infty dpq_2(\rho) E_{02}. \quad (4)$$

where the integration is performed at $x=d+\Delta$. Hence, it is evident that the odd radiation modes created by the scattering are responsible for the transverse force on the waveguide. A direct measure of the waveguide’s tendency to move either way is the derivative of the force with respect to $\Delta$, $dF_{x2}/d\Delta (\Delta = 0)$ denoted by $F'_{x2}$. Bearing in mind that $F_{x2}(\Delta = 0) = 0$, the force may be approximated by $F'_{x2} \Delta$. The only term in the above equation that depends on $\Delta$ is $q_{02}$, and an analytic expression for $dq_{02}/d\Delta$ may be obtained. The integration over $z$ is then performed analytically, and the closed-form expression for the derivative reads

$$F_{x2} = \frac{1}{4 \varepsilon_0 (\varepsilon_i - 1)} \int_0^\infty dz z [E_2^2]_{z=d+\Delta} - [E_2^2]_{z=-d+\Delta}. \quad (5)$$

where $\sigma = \sqrt{\varepsilon_0 k_0^2 - \beta_i^2}$ is the radiation modes transverse wave number inside the slab. While the spectrum of radiation modes contains both propagating modes having real $\beta_i$ (where $\rho < k_0$) and evanescent modes having imaginary $\beta_i$ (where $\rho > k_0$), the expression in the square brackets is purely imaginary for the propagating modes, so that it is only the evanescent radiation modes that contribute to the generation of this force.

Both the force $F_{x2}$ [Eq. (4)] and the quantity $F'_{x2} \Delta$ [Eq. (5)] are plotted in Fig. 2(a) as a function of $d$ for an offset $\Delta$ of 2% of $d$. The range of $d$ shown is 20–110 nm, where the slab is single mode in each polarization. The wavelength is taken to be $\lambda = 1.55 \mu m$ and the permittivity is 3.48$^2$, corresponding to silicon at that wavelength. The forces are normalized by $F_0 = P_m/c$, which is the momentum per unit time carried by a plane wave. These results are compared with a finite element method (FEM) simulation, and as seen the three curves are virtually identical. In addition to the integration of the force on the polarization densities, all results were validated by integration over the Maxwell stress tensor [21] with excellent agreement.

The plot of Fig. 2(a) reveals two regimes: the first is a self-alignment regime up to a slab half-width of about 57 nm, for which $F'_{x2} \Delta < 0$ corresponding to a restoring force. The
second is an instability regime in which a small offset results in a deflection force. Although we show here only the transverse force on the output waveguide, when the offset is small, the scattered power is negligible and by virtue of momentum conservation, the force on the input waveguide is of the same magnitude and opposite in sign to that on the output waveguide. In fact, we have shown analytically that

\[ F_1 = -F_2. \]

To better illustrate the different regimes, we depict in Fig. 3(a) the transverse force \( F_{x2} \) as a function of \( \Delta \) for \( d=39 \) nm where a negative restoring force is seen, \( d=57 \) nm where the force derivative at \( \Delta=0 \) vanishes at the transition between the two regimes, and for \( d=110 \) nm where instability in the form of a deflecting force is observed.

When the incident mode is transverse magnetic (TM), the situation is considerably more involved. The field components for the TM mode are \( E_z \), \( E_r \), and \( H_r \), and the incident magnetic field is given by \( H_i = \alpha_{TM} \cos(k_r x) \exp(-j\beta_z z) \) [20]. The fields in each region are described by Eqs. (1) and (2) with \( E \) replaced by \( H \). The force mechanism in the TM case differs substantially than that of the TE case, as for the TM the electric field has normal components to the discontinuity in the dielectric, and therefore polarization surface charge densities are formed on the top and bottom surfaces of the waveguide and from the polarization volume current density. As shown below, these two contributions are subdominant in the region of interest. Similarly to the TE mode, only the evanescent part of the odd radiation mode spectrum participates in the generation of the force as the expression in the curly brackets, as well as \( \alpha_{SM} \), is purely real for \( \rho<k_0 \). The relation \( F_1 = -F_2 \) holds for the TM case as well.

The quantities \( \alpha_{SM} \), \( \alpha_{TM} \), as well as the force calculated by FEM, are shown in Fig. 2(b) as a function of \( d \) for an offset \( \Delta \) of 2%. Contrary to the TE case, here instability occurs for small values of \( d \), and above about 70 nm there is self-alignment. The instability peak force for the TM is weaker by about an order of magnitude than that of the TE. The restoring force, however, increases monotonically, so that at the maximum value in the shown range of \( d \), it is about two orders of magnitude stronger than the peak value of the TE restoring force. This dramatic difference is due to the presence of electric field components that are perpendicular to the dielectric boundaries. Specifically, at the left boundary of the output waveguide (\( z=0 \) plane), \( E_z \) induces a polarization surface charge density, while \( E_z \) gives it a transverse kick. The result \( F_{x2,1} \) given by Eq. (6) is plotted in Fig. 2(b). This force is negative for the entire range of \( d \) and is seen to comprise almost all of the total force in the self-alignment regime. Qualitatively, it may be associated with a dipole induced by the guided mode, which has \( E_z = \sin(k_d d) \), and the strength of the dipole per power increases with \( d \) as the mode confinement increases. At the \( z=0 \) interface, the dipole is roughly inverted, and, consequently, the two parts attract each other, in both the transverse and longitudinal directions. Moreover, this force grows rapidly as a function of \( \Delta \) and is therefore responsible for the derivative approximation being less accurate than for the TE case. This is seen in Fig. 3(b), where \( F_2 \) is plotted for four different values of \( d \): \( d=35 \) nm where instability is observed, \( d=70 \) nm about where the derivative vanishes at \( \Delta=0 \), \( d=85 \) nm which exhibits self-alignment, and \( d=110 \) nm, where there is strong self-alignment which is further discussed below.

We next extend the discussion by introducing a longitudinal gap \( g \neq 0 \). Figures 4(a) and 4(b) show contours of \( F_{x2} \) in the \( g - \Delta \) plane obtained by FEM simulations, for TE and TM incident modes, respectively. The slab half-width is assumed

\[ \frac{F_{x2}}{F_0} = \begin{cases} \frac{2}{(\omega\epsilon_0\epsilon_r)^2} \int_0^{\infty} dp q_{x2}(p) \\ \times \left( \frac{(k_0 \beta_z + \sigma \beta_r) \sin(k_0 - \sigma d)}{(k_0 - \sigma)} \right) \\ - \left( \frac{(k_0 \beta_z - \sigma \beta_r) \sin(k_0 + \sigma d)}{(k_0 + \sigma)} \right) \end{cases} \]
to be $d=110$ nm, where according to Fig. 2, the TE mode places the system in instability, whereas the TM mode causes self-alignment. In both cases, the $g=0$ behavior extends to larger values of $g$, but the decay of the TM repulsive force with the offset and gap is much slower than that of the attractive TM force, as seen by the scales of $g$ and $\Delta$ of the two frames of Fig. 2. The maximum TE force is about $0.4F_0$ and it is obtained for about $\Delta = 130$ nm and $g = 50$ nm. A waveguide cantilever at $\Delta = 0$ is, in fact, in a bistable state, where a small offset may result in a deflection force that would eventually be balanced by the mechanical force. The maximum attractive TM force is obtained for $g=0$ and $\Delta = 45$ nm, and it is about $0.7F_0$. We further found that a strong longitudinal force, facilitated mainly by the $E_0$ dipoles at the two facets, is pulling the two waveguides toward each other at a force of about $2F_0$ for $g=20$ nm. For large enough values of $g$, the TM force becomes repulsive, corresponding to radiation pressure.

The self-alignment and instability may be tested experimentally by fabricating on a chip two waveguide cantilevers with an offset and a gap. For instance, taking a length in the $y$ direction $\Delta_y$, then for $P_{in}\Delta_y = 30$ mW, $F_{0}\Delta_y = 100$ pN, and at $\Delta = 50$ nm and $g = 20$ nm, we obtain $F_0\Delta_y = -0.1F_0\Delta_y$ according to Fig. 2(b), which is about 10 pN. This is on the order of magnitude of force that was shown in Ref. [18] to actuate a silicon suspended waveguide, and similarly the thermal effects are expected to be insignificant compared to the optical forces. Three-dimensional FEM simulations show that for the fundamental mode of a rectangular waveguide 400 nm wide and 200 nm thick, the maximum force is enhanced, so that at $g=20$ nm, it is about $-0.3F_0$ obtained at $\Delta = 100$ nm. The deflection of the waveguide in such a system may be viewed by the nonlinear input-output behavior, as the output power increases when the cantilevers tend to self-align. The cantilevers may also be vibrated at their mechanical resonance by modulating the incident power, resulting in a system that may be suitable for applications such as sensing.

In conclusion, we demonstrated an effect of light forces in the form of self-alignment or instability of a waveguide broken by an offset and a gap. The waveguide size and mode polarization determine which of the two regimes the waveguide is in. Closed-form expressions for the transverse forces were given for the case of a small offset and no gap. The forces described here are unique as they are due to the presence of both the guided mode and the scattered light from the discontinuity. Strong self-alignment in both dimensions for a TM mode is caused by near field interaction of the polarization surface charges created by the longitudinal electric field. The effects presented here suggest a way for holding a cantilever in a stable equilibrium as well as vibrating it, with applications in light-driven machines, and specifically in sensing.

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