

Design of optimized dispersive resonant cavities for nonlinear wave mixing

Guy Klemens, Chyong-Hua Chen, Yesaiahu Fainman

Department of Electrical and Computer Engineering
University of California, San Diego
9500 Gilman Drive
La Jolla, California 92093-0407
gklemens@ucsd.edu

Abstract: Dispersive mirrors can be designed to create cavities that resonate at set multiple frequencies while simultaneously meeting the conditions for efficient nonlinear wave mixing. We analyze the conditions that such a cavity design must meet and the free parameters that can be used for optimization. Using numerical methods, we show the benefit in conversion efficiency attained with multiple resonances, and draw conclusions concerning the design parameters. As a specific example, we consider parametric downconversion in a triply-resonant cavity.

© 2005 Optical Society of America

OCIS codes: (190.4360) Nonlinear Optics, devices; (190.2620) Frequency conversion; (230.3990) Microstructure devices

References and links

1. J.A. Armstrong, N. Bloembergen, J. Ducuing and P. S. Pershan, "Interactions between light waves in a nonlinear dielectric," *Phys. Rev.* **127**, 1918–1939 (1962).
2. A. Ashkin, G. D. Boyd, and J. M. Dziedzic, "Resonant optical second harmonic generation and mixing," *IEEE J. Quantum Electron.* **QE-2**, 109–124 (1966).
3. I. Shoji, T. Kondo, A. Kitamoto, M. Shirane and R. Ito, "Absolute scale of second-order nonlinear-optical coefficients," *J. Opt. Soc. Am. B* **14**, 2268–2294 (1997).
4. V. Berger, "Second-harmonic generation in monolithic cavities," *J. Opt. Soc. Am. B* **14**, 1351–1360 (1997).
5. C. Simonneau, *et al.*, "Second-harmonic generation in a doubly resonant semiconductor microcavity," *Opt. Lett.* **22**, 1775–1777 (1997).
6. F. F. Ren, *et al.*, "Giant enhancement of second harmonic generation in a finite photonic crystal with a single defect and dual-localized modes," *Phys. Rev. B* **70**, 245109 (2004).
7. R. Haidar, N. Forget, and E. Rosencher, "Optical parametric oscillation in microcavities based on isotropic semiconductors: a theoretical study," *IEEE J. Quantum Electron.* **39**, 569–576 (2003).
8. C. H. Chen, K. Tetz, W. Nakagawa, and Y. Fainman, "Wide-field-of-view GaAs/Al_xO_y one-dimensional photonic crystal filter," *Appl. Opt.* **44**, 1503–1511 (2005).
9. W. Nakagawa, P. C. Sun, C. H. Chen, and Y. Fainman, "Wide-field-of-view narrow-band spectral filters based on photonic crystal nanocavities," *Opt. Lett.* **27**, 191–193 (2002).
10. E. G. Sauter, *Nonlinear Optics*, pp. 55, John Wiley (1996).
11. M.M. Fejer, G.A. Magel, D.H. Jundt, and R.L. Byer, "Quasi-Phase-Matched Second Harmonic Generation: Tuning and Tolerances," *IEEE J. Quantum Electron.* **28**, 2631–2654 (1992).
12. M. Born and E. Wolf, *Principles of Optics, Seventh Edition*, pp. 360, Cambridge (1999).
13. A. Yariv, *Quantum Electronics, Third Edition*, pp. 147, John Wiley (1989).
14. W. J. Tropf, M. E. Thomas, and T. J. Harris, "Properties of crystals and glasses," in *Handbook of Optics: Volume II*, McGraw-Hill (1995).
15. D.S. Bethune, "Optical harmonic generation and mixing in multilayer media: analysis using optical transfer matrix techniques," *J. Opt. Soc. Am. B* **6**, 910–916 (1989).
16. K. L. Vodopyanov, *et al.*, "Optical parametric oscillation in quasi-phase-matched GaAs," *Opt. Lett.* , **29** 1912–1914 (2004).

1. Introduction

Resonant cavities have long been used to increase the efficiency of nonlinear processes, particularly second-harmonic generation (SHG). The idea was first described in [1], whereby achieving resonance at the fundamental frequency would raise the input field magnitudes, and designing the cavity to resonate at the harmonic frequency would increase output power by multiple passes. This is conceptually similar to quasi-phase matching (QPM), with the periodically inverted domains collocated in the cavity. This concept was further explored both theoretically and experimentally in [2]. In that paper, resonance occurred either at the fundamental frequency or at the harmonic frequency, but not at both due to limitations in mirror design. The introduction of photonic-crystal mirrors has allowed for the design of cavities that resonate at predetermined frequencies, making doubly-resonant cavities possible. This method is particularly useful for isotropic nonlinear materials such as gallium arsenide (GaAs), for which other phase matching methods are difficult to achieve. Gallium arsenide offers a high second-order nonlinear coefficient of approximately 100 pm/V [3], but is limited in its use for wavelength conversion by the difficulty in achieving phase-matching. Several designs and analyses have been presented showing the enhancement of SHG with GaAs in doubly-resonant cavities [4, 5, 6]. Unlike SHG, conversion to a lower frequency is achieved through a three-wave parametric process involving three separate frequencies. A design using a doubly resonant cavity with two sets of Bragg mirrors was described in [7], although a triply-resonant structure has not yet been presented.

The flexibility given by designs of 1-D and 2-D dielectric resonant structures [8, 9] allows for the improvement of nonlinear wave mixing efficiency beyond what has been previously analyzed. In this manuscript we investigate the Dispersive Resonant Cavity (DRC) as a way to satisfy all of the required conditions for efficient nonlinear wave mixing and to maximize the output efficiency. We describe the necessary conditions as well as the degrees of freedom available in the design, such as cavity length and the amplitude and phase of the cavity mirror reflection coefficient. The contribution of this manuscript is the use of dispersive resonant structures to achieve optimized quasi-phase matching in nonlinear wave mixing processes. Finally, we present a numerical example of a triply-resonant cavity of GaAs used for downconversion, in order to demonstrate the effects of the various parameters on a design.

2. Dispersive resonant cavity for efficient nonlinear wave mixing

The resonant cavity design methods discussed in the introduction have not made use of recent advancements in the design of dispersive dielectric mirrors with arbitrary reflection coefficients. A general configuration of such a structure is shown in Fig. 1. Two dielectric mirrors are used to create multiple Fabry-Perot-type cavities operating at multiple optical frequencies. Wave mixing occurs in the nonlinear material placed between the two mirrors. For efficient nonlinear wave interaction, two conditions must be simultaneously satisfied: (i) conservation of momentum, and (ii) conservation of energy. When the nonlinear conversion is performed in a cavity, the cavity parameters must be designed so that the cavity is either transparent or resonant, since an off-resonance Fabry-Perot cavity would contain little or no pump beam energy. We consider here cavities that are designed to resonate at all the relevant frequencies of the nonlinear process, which adds another design condition: (iii) cavity resonance requirements.

Consider a process involving P frequencies, the differential equation describing the evolution of the complex envelope of each wave q , with $q = 1 \dots P$, is [1]

$$\frac{dA_q(z)}{dz} = -i \frac{\omega_q \chi_{eff}}{c_0 n_q} \prod_{p \neq q}^P A_p(z) \exp(i\Delta k z) \exp(i\Delta\omega t), \quad (1)$$

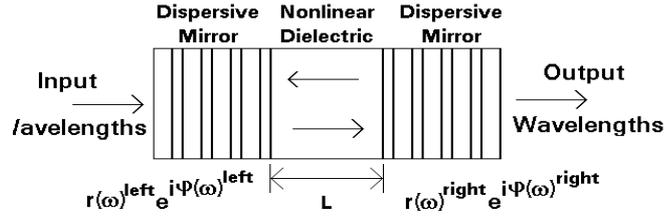


Fig. 1. Diagram of a multiply-resonant cavity used for nonlinear wavelength conversion.

where A_p are the complex envelopes, ω_q are the frequencies, χ_{eff} is the effective nonlinear coefficient, c_0 is the speed of light, n_q are the refractive coefficients, Δk is the phase-mismatch term and $\Delta\omega$ is the frequency mismatch term. We assume nondegenerate wave mixing here, although the analysis still applies in the degenerate case with two or more waves having the same frequency. We define the phase-mismatch by

$$\Delta k = \sum_r^R k_r - \sum_s^S k_s, \quad (2)$$

where the first summation is made over the wavenumbers of the R input photons in the process, and the second summation is made over the wavenumbers of the S output photons, with $S + R = P$. The wavenumber at frequency ω_q in a medium of refractive index n_q is defined as $k_q = \frac{\omega_q n_q}{c_0}$. Some of the complex envelopes may be conjugated in Eq. (1), depending on the specific process. We define the frequency mismatch term as

$$\Delta\omega = \sum_r^R \omega_r - \sum_s^S \omega_s, \quad (3)$$

where ω_r are the frequencies of the input photons and ω_s are the frequencies of the output photons.

2.1. Phase matching

Conservation of momentum is equivalent to satisfying the phase matching condition. When $\Delta k = 0$, phase match is achieved and the conversion can continue until the input beams are fully depleted. When $\Delta k \neq 0$, the photons generated along the propagation direction will cancel the output signal energy, coupling it back to the input fields. For example, in a three-wave mixing process, the output power will follow a $\sin^2(x)$ curve [1], with the distance between zero points being $2\pi/\Delta k$, defined as the coherence length [10], L_c . (Note that some references, e.g., [11], define the coherence length as $L_c = \pi/\Delta k$.) In our design we use the phase of the reflection of the cavity mirrors to compensate for the phase-mismatch at multiple frequencies and we explore the advantages of this method over existing approaches. The phase compensation condition is expressed as

$$2\Delta k L + \sum_p^P \left(\varphi_p^{(left)} + \varphi_p^{(right)} \right) = 2\pi a^{(1)}, \quad (4)$$

where L is the cavity length, $a^{(1)}$ is an integer, and $\varphi_p^{(left)}$ and $\varphi_p^{(right)}$ are the mirror phases at the left and right side of the cavity, respectively, at frequency p . We define the complex mirror reflection coefficient

$$r(\omega) = |r(\omega)| \exp(i\varphi(\omega)). \quad (5)$$

2.2. Energy conservation

Conservation of energy determines the frequencies involved in the nonlinear wave mixing. The condition $\Delta\omega=0$ applies to every nonlinear process at every point in the cavity. Unlike phase-matching, this condition can not be compensated by the mirrors.

2.3. Cavity resonance condition

The cavity design must also satisfy the Fabry-Perot resonance condition at each frequency, leading to the expression

$$2k_p L - \phi_p^{left} - \phi_p^{right} = 2\pi a^{(2)}, \quad (6)$$

where $a^{(2)}$ is an integer, and p varies through all the frequencies involved. If Eq. (6) is not satisfied for a given frequency, destructive interference will minimize the field strength in the cavity. In applications such as [2], the mirrors at some frequencies have negligible reflection coefficients to allow those fields to pass through the cavity, thereby bypassing the resonance condition. We use the added flexibility of dispersive dielectric mirrors to apply the resonance condition to all the frequencies involved in the nonlinear wave mixing process. Eq. (6) applied to each frequency, along with Eq. (4) can be solved to find values for the mirror reflection phases.

For efficient nonlinear wave mixing processes in a dispersive resonant cavity, these three conditions must be satisfied simultaneously. In the following we explore the degrees of freedom in the design parameters of dispersive resonant cavities to meet these conditions and to optimize the output efficiency.

3. Parameters in the design of dispersive resonant cavities

There are two degrees of freedom available in the design of dispersive resonant cavities for nonlinear wave mixing: (i) cavity length, and (ii) amplitude and phase of the mirror reflection coefficients. The mirror reflection coefficients are used to set the resonance condition for each frequency, determining the cavity finesse at each frequency. The cavity length can be optimized to satisfy the phase matching condition and maximize output power. The importance of these parameters, as well as their optimal values, can be determined by considering Eq. (1).

The rate of the nonlinear conversion can be seen in Eq. (1) to be directly related to the magnitude of the input fields. A monochromatic wave of amplitude $A_{Incident}$ incident upon a cavity that resonates at the wave's frequency will create a field inside the cavity of magnitude [12]

$$A_{Internal} = \frac{\sqrt{1-R}}{1-\sqrt{R}} A_{Incident}, \quad (7)$$

where R is the reflection coefficient of the cavity mirrors, $R = |r(\omega)|^2$. The larger field amplitudes of the input beams increases the rate of growth of the output beam, thereby increasing overall output efficiency. Furthermore, the output beam benefits from multiple passes through the cavity. The number of interfering beams corresponds to the cavity finesse, \mathcal{F} , which is related to the mirror reflectivity by [12]

$$\mathcal{F} = \frac{\pi\sqrt{R}}{1-R}. \quad (8)$$

Alternatively, the increased interaction time can be quantified by the cavity lifetime, which is the time taken for a pulse in the cavity to decay to e^{-1} times its initial value. The cavity lifetime, t_c , is related to the finesse by [13]

$$t_c = \frac{nL}{\pi c_0} \mathcal{F}, \quad (9)$$

where n is the refractive index and L is the cavity length. From the reasons given above, the conversion efficiency will be higher when the cavity finesse is high at the input frequencies and the cavity lifetime is long for the output frequencies. While the cavity length is part of Eq. (9), this parameter can not be arbitrarily made larger to maximize output power.

The cavity length significantly affects the output signal efficiency, and any length beyond the optimal value either has no effect or decreases conversion efficiency, depending on the exact length used. A solution of coupled-wave equations based on Eq. (1) shows that the power in the signal beam follows a $\sin^2(x)$ relation as the beams propagate through a crystal [1], with the zero points a coherence length, $2\pi/\Delta k$, apart. A pulse at the output wavelength passes approximately \mathcal{F} times through the cavity, leading to an approximation for output power of the form

$$P_{out} = C\mathcal{F} \sin^2\left(\frac{\pi}{L_c}L\right), \quad (10)$$

where L_c is the coherence length and C is a constant dependent on the nonlinear constant of the cavity dielectric, the input field magnitudes within the cavity, and the transmission of the mirrors. From this equation we deduce that a cavity length of one coherence length will yield no output power, since the converted power will have all reverted back to the input waves. Furthermore, Eq. (10) is periodic, so any cavity length beyond one coherence length is redundant. The maximum output power occurs when $L = L_c/2$. For SHG, this is the point at which the fundamental and the harmonic fields are 180 degrees apart in phase, and can be matched with the domain inversion of periodic poling [11]. A more detailed analysis can be performed using an approximate numerical method on a specific case, as is done in the next section.

4. Case study: Downconversion in a triply-resonant GaAs dispersive resonant cavity

4.1. Numerical Estimation of Nonlinear Conversion

As an example, we present a DRC for efficient downconversion in GaAs. We assume conversion in GaAs with pump, signal and idler frequencies corresponding to wavelengths of 1.55, 3.8 and 2.617 micrometers (μm), respectively. From the Sellmeier dispersion equations [14], the refractive indices at these frequencies are 3.3989, 3.3263 and 3.3427, respectively. Specific mirror design using multilayer design methods is discussed in Section 6.

To estimate nonlinear conversion efficiency for multiply-resonant structures, we use the transfer matrix technique [15]. The dielectric interfaces are represented by electric field transfer matrices, which are combined to find the overall reflection, transmission and internal fields within the structure. After calculating the field magnitudes of the pump and idler within the cavity, the nonlinear polarization, P_{NL} , is found through the definition

$$P_{NL} = \chi^{(2)}E_pE_i^*, \quad (11)$$

with E_p and E_i representing the electric field magnitudes of the pump and idler, respectively, and $*$ indicating complex conjugation. The nonlinear coefficient, $\chi^{(2)}$, is dependent upon the material and the direction of propagation relative to the crystal structure. This nonlinear polarization is then used as a source distributed within the cavity, and the transfer matrix method is used to find the resulting output field. This is the method used for the calculations presented below. As an additional step, if the signal field magnitude is not small in comparison to the pump and idler fields, this procedure can be iterated to find the pump-depleted solution.

5. Design parameters for specific design case

The conclusions of Section 3 can be numerically demonstrated for the example case. One design parameter available for optimization is cavity length. Unlike the domain inversions of QPM,

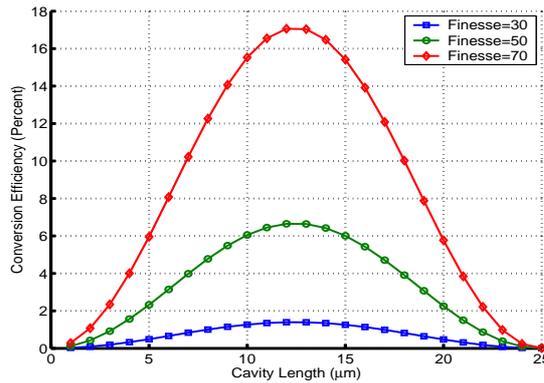


Fig. 2. The calculated conversion efficiency as a function of cavity length for a dispersive resonant cavity. Input wavelengths of 1.55 and 2.617 micrometers (μm) and an output wavelength of 3.80 μm were assumed, with GaAs as the nonlinear material. The corresponding coherence length is 25 μm . The cavity finesse is held constant at 30, 50 and 70, and the input field intensities are $0.5 \text{ W}/\mu\text{m}^2$.

which can only produce a phase shift of 180 degrees, the mirrors of a DRC can be designed to produce a wide range of phase shifts. Cavities of arbitrary lengths can therefore be used. Upon consideration of the physical mechanisms of wavelength conversion along with the effects of the resonant cavity, conclusions can be drawn concerning optimal cavity length. As described in Section 2, the solution of Equation 1 follows a $\sin^2(x)$ curve, with the zero points a coherence length apart. Any cavity length beyond half of a coherence length is not optimal since there will be reconversion of signal energy to the input beams within the cavity. There is therefore less energy conversion per pass. For cavity lengths smaller than half of a coherence length there is no reconversion depleting the signal beam. The conversion approaches the phase matched case as the cavity becomes smaller; however, the effective cavity length of $\mathcal{F}L$ approaches zero, leading to less conversion. For a cavity length that is very small with respect to the coherence length, the rate of nonlinear conversion would be high, but the finesse would have to also be high in order to have enough effective interaction length for efficient conversion. The estimated conversion efficiency as a function of cavity length for the example case and mirrors described in Section 6 is shown in Fig. 2. The optimal cavity length is at $L = L_c/2$, which is similar to quasi-phase matching. This conclusion applies independently of the cavity finessses. The DRC has advantages, however, over QPM. While the signal beam reflecting within the cavity is similar to propagation across multiple poled domains, the input beam field magnitudes are increased in the cavity, further increasing conversion efficiency. Furthermore, since only one cavity length must be fabricated for the DRC, the tolerances involved in creating dozens or hundreds of adjacent inverted domains is avoided.

The advantage of designing a multiply-resonant structure is shown in Fig. 3. The output conversion efficiency as a function of input power is shown for cases of singly, doubly and triply resonant structures. Each additional resonance increases the efficiency by approximately an order of magnitude, demonstrating the advantage of a multiply-resonant structure over a structure without resonances at each relevant frequency.

The relative significance of the finesse at the three frequencies is examined in Fig. 4. In Fig. 4(a), the idler finesse is held constant at 50 while the pump and signal finesse are varied, while in Fig. 4(b), the pump finesse is held constant at 50 while the idler and signal finesse are varied.

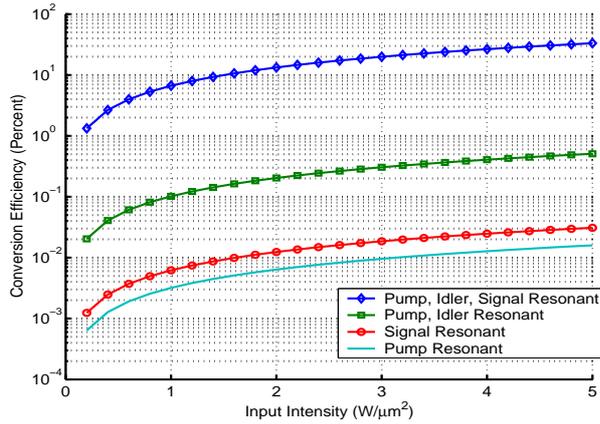


Fig. 3. The calculated conversion efficiency as a function of input power. Input wavelengths of 1.55 and 2.617 micrometers (μm) and an output wavelength of 3.80 μm were assumed, with GaAs as the nonlinear material. Ideal phase compensating mirrors were assumed, and the cavity length was held constant at 12 μm . Shown are curves for cases where (i) the pump, (ii) the signal, (iii) the pump and idler, and (iv) the pump, idler, and the signal are resonant. For the resonant beams, the finesse was set to 50, and was set to 0 for the non-resonant beams.

As expected from the symmetry between pump and idler power in Eq. (1), the finesse of the cavity at the pump and idler frequencies are of equal importance. In Fig. 4(c), the pump and idler are given equal finesse, which varies along with the signal finesse. These figures show the signal finesse to be of greater importance than the pump or idler finesse. This is because the short physical cavity length increases the importance of the effective cavity length $\mathcal{F}L$. For designs such as this, therefore, the reflectivity of the mirrors at the signal frequency can be increased at the expense of the reflectivities at the pump and idler frequencies.

6. Mirror design

For efficient conversion, the cavity mirror reflection coefficients should be designed to simultaneously satisfy Eq. (4) and Eq. (6) for each frequency. Also, the magnitudes of the reflection coefficients of the mirrors should be relatively high at each frequency. These conditions are met for the dual-resonant design of [7] with two sets of dielectric stacks, each meeting the Bragg condition at one wavelength. Instead, we follow the method of [4]-[6] by using an aperiodic stack of dielectric layers to meet the required conditions. This gives more flexibility in the design of the mirror reflection coefficients. The reflection and transmission coefficients of a stack can be found using standard transmission-matrix methods. Circuit optimization programs can also be used with ideal transmission line segments representing the dielectric layers. The characteristic impedance of each transmission line segment is set to $1/n$, where n is the refractive index, which can be given by a dispersion equation. The dielectric constant of the transmission line is set to n^2 . Ports with impedance of 1 Ohm (assuming a vacuum surrounding medium) can be placed at both sides of the cavity, and the optimizer run to maximize power transmission from port 1 to port 2 at the frequencies of interest. As an additional optimization parameter, a port with the input impedance of the cavity material can be attached to a single mirror with a 1 Ohm resistor placed at the outside of the mirror to represent the surrounding medium. The reflection seen at that third port is maximized while the transmission between the first two ports is

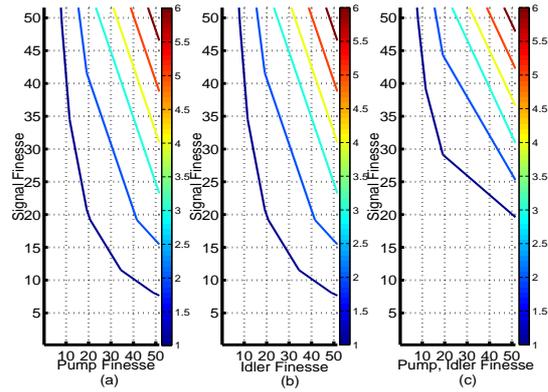


Fig. 4. The calculated conversion efficiency as a function of cavity finesse. Input wavelengths of 1.55 and 2.617 micrometers (μm) and an output wavelength of 3.80 μm were assumed, with GaAs as the nonlinear material. The cavity length is 12 μm and the input field intensities are $0.5 \text{ W}/\mu\text{m}^2$. (a) Idler finesse held constant at 50 while signal and pump finesse vary, (b) pump finesse held constant at 50 while signal and idler finesse vary, (c) pump and idler finesse are set to be equal and vary together vs signal finesse.

also maximized. This method has the advantage that there is a wide variety of circuit simulators and optimization routines available.

We found a set of 44 layer thicknesses with the necessary reflection characteristics, as shown in Fig. 5. The layers were alternating between GaAs and aluminum arsenide (AIAs). The refractive indices of AIAs at the pump, signal and idler are 2.909, 2.886 and 2.880. The layer thicknesses making up this mirror are shown in Fig. 6. The mirror reflectivities at the three wavelengths are approximately $R = 0.95$, corresponding to a finesse of 50 and a cavity lifetime of 2.2 psec. The estimated conversion efficiency is approximately 5 %. We stress that this is achieved with a nonlinear dielectric length of only 12 micrometers. This was the maximum finesse that could be reached with these parameters. More layers are necessary to achieve a higher finesse. Another possible method for increasing conversion efficiency is to combine a few domains of periodically-poled GaAs, as in [16], within a DRC.

The yield of useable devices is limited by the tight tolerance requirements on the mirror parameters. The frequencies of the transmission peaks shown in Fig. 5 are determined by the mirror reflection phases, and are therefore especially sensitive to parameter variations. Monte-Carlo simulation of the 44-layer design, for example, shows that a random variation of up to 5 % in each layer thickness can shift the transmission peaks by approximately 1 % in wavelength. Increasing the number of layers in order to increase the cavity finesse will narrow the transmission peaks, further limiting the parameter tolerances. The movement of the transmission peaks directly affects the conversion efficiency. In one example, all of the layers were randomly varied in thickness by up to 10 %, moving the transmission peaks. The calculated conversion efficiency using the design values of 1.550, 2.617 and 3.800 μm is less than .1 %. If the wavelengths used are adjusted to 1.552, 2.621 and 3.799 μm , the calculated conversion efficiency is approximately 1 %. Note that the three frequencies of the transmission peaks do not necessarily satisfy Eq. (3), lowering the device yield.

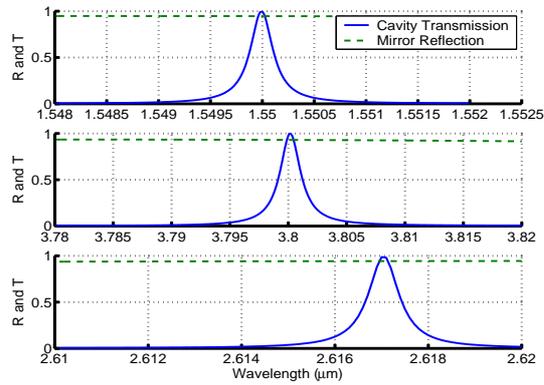


Fig. 5. Transmission coefficient (solid line) through the entire structure and reflection coefficient (dashed line) seen from the inside of the cavity looking out through the dielectric stack. At the three wavelengths of the nonlinear conversion, the transmission is at a peak, indicating Fabry-Perot resonance. The mirror reflectivities are at a maximum at those wavelengths to increase cavity finesse.

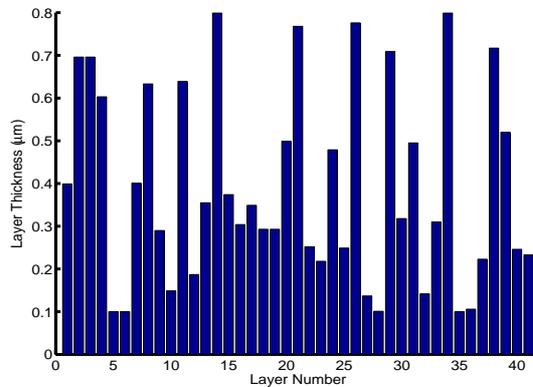


Fig. 6. Layer thicknesses of alternating GaAs and AlAs for the dielectric mirrors with the response of Figure 5. Layer 1 is GaAs. The refractive indices used are 3.399, 3.343, and 3.326 for GaAs at 1.55, 2.617 and 3.8 μm , respectively, and 2.909, 2.886 and 2.880 for AlAs.

7. Conclusion

We have examined the design of resonant dielectric cavities for nonlinear wave mixing. These structures are capable of simultaneously satisfying conservation of energy, the phase matching condition and the Fabry-Perot conditions for multiple frequencies. Unlike QPM, the mirrors are not restricted to phase shifts of 180 degrees, allowing for variation in the cavity length from half of a coherence length. As the cavity length decreases, the power conversion approaches the phase matched case; however, the effective cavity length of $\mathcal{F}L$ also becomes smaller, lowering the conversion efficiency. The trade-off reaches an optimal point at a cavity length of $L_c/2$, where the output power is maximized. As a design example, we presented a triply-resonant cavity made up of GaAs with dielectric mirrors made of alternating layers of GaAs and AlAs. While the conversion efficiency is not higher than in other implementations, the device volume is small, using a 12 μm dielectric length. Although the layer thickness tolerances are strict, this design could be fabricated with with current processes, such as Metal Organic Chemical Vapor Deposition.