Nonlinear Fabry–Perot resonator with a silicon photonic crystal waveguide

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We derive an equation that describes the nonlinear operation of a Fabry–Perot resonator with a large group index waveguide. Specifically, a silicon photonic crystal microcavity with two-photon-excited free carrier nonlinearity and Kerr nonlinearity is assumed. The equation clearly explains the bistability of the device and the reduction of the required pump energy for a specific nonlinear phase shift at an appropriate phase detuning from the resonance. We present a simple procedure to predict the required optical pump energy for the modulation and the resulting modulation depth by use of the equation and the device parameters.

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Silicon photonics has become an active research area due to its compatibility with standard complimentary metal-oxide semiconductor technology, resulting in demonstration of various silicon-based devices such as modulators, lasers, and switches. Since the nonlinear effects in silicon are very small, field localization using resonant nanostructures is being developed to enhance the efficiency of nonlinear optical processes. Specifically, ultrahigh-quality factors that have been demonstrated in silicon photonic crystal (PhC) microcavities are attractive for enhancing the efficiency of nonlinear interactions in silicon-based devices. However, to our knowledge detailed analysis of resonant cavities realized with PhC lattices has not yet been sufficiently advanced. In this Letter, we introduce a simple analysis of the nonlinear phase shift in a Fabry–Perot resonator realized with a two-dimensional (2D) PhC waveguide in silicon. For given linear parameters of a resonator, we present an analytic procedure to predict the optical pump energy required to achieve nonlinear operation of a Fabry–Perot resonator with the desired modulation depth.

Consider a simple model described in the diagram in Fig. 1, where the two mirrors with amplitude reflectance \( r \), amplitude transmittance \( t \), and amplitude scattering loss \( s \) are placed at \( z = 0 \) and \( z = L \), respectively. We assume that the waveguide between them has material index \( n \) and corresponding group index \( n_g \) (equivalently group velocity \( v_g \)). We also assume that the refractive index \( n \) of the material changes due to the nonlinear Kerr effect, \( n_g J \), and to the Drude effect via free carrier generation by two-photon absorption (TPA), \( n_d I^2 \):

\[
\begin{align*}
n & = (n_0 - jn_j) + n_d I^2 + (n_2 - jn_{2j})I, \\
n_d & = -\frac{e^2}{2n_0 a_0 m^* e_0 4 \ln 2 2 \hbar \omega_p},
\end{align*}
\]

where \( n_0 \) and \( n_j \) are the real and the imaginary part of the linear refractive index of the waveguide material, respectively; \( n_2 \) and \( n_{2j} \) are the real and imaginary part of the nonlinear refractive index due to Kerr nonlinearity, respectively. Assuming a Gaussian-shaped pump pulse with full width at half-maximum (FWHM), \( \tau_p \), and angular frequency \( \omega_p \), the value of \( n_d \) can be expressed for a time scale much shorter than the carrier lifetime as:

\[
n_d = -\frac{e^2}{2n_0 a_0 m^* e_0 4 \ln 2 2 \hbar \omega_p},
\]

where \( e \) is the electron charge, \( \omega_p \) is the angular frequency of the signal, \( m^* \) is the effective mass of an electron, \( n_0 \) is the vacuum permittivity, and \( \beta \) is the TPA coefficient. The propagation constant for the field inside the waveguide shown in Fig. 1 is given by:

\[
k = k_0 n = k_0 \frac{n}{\cos \theta'},
\]

where \( k_0 \) is the free-space wavenumber and \( \theta \) is the angle between the waveguide direction and the direction of the phase velocity. The zigzag ray picture for the group index is valid for the strongly confined waveguide, where the effect of Goos–Hänchen shift can be neglected, which will be the case for a 2D PhC waveguide when the small dispersion from material and vertical index guiding is neglected. Substituting Eq. (1) into Eq. (3) yields:

\[
k = k_0 \frac{\alpha}{\cos \theta' + \beta I} = \kappa - j \mu,
\]

where \( \alpha = 2k_0 n_{\theta j} \) is the attenuation coefficient that includes losses due to single-photon absorption and waveguide loss, and \( \beta = 2k_0 n_{\mu j} \) is the TPA coefficient already defined. Here, we also defined \( \kappa \) and \( \mu \) as the real and the imaginary part of the propagation constant, respectively. A large value of \( n_j \) (or large \( \theta \) cannot be achieved using a PhC waveguide, and we apply our theory to the PhC microcavity later. We find the intensity of the field inside the cavity, \( I(z) \), normalized to the intensity of the input field, \( |a|^2 \).
Fig. 1. Schematic diagram describing the analytic model for Fabry–Perot resonator in a large group index waveguide.

\[
I(z) = \frac{1}{|a|^2} e^{-2i\xi} + \frac{R}{1 - 2Re^{-2i\xi} \cos 2(\phi(L) - \phi(z))} R^2 e^{-4iL} + 2Re^{-2i\xi} \cos 2(\phi(L) - \phi(z)) \]
\[
\times (1 - S^2 - R^2),
\]

where we assumed real values for the reflectance, \( R = R \), and the scattering loss, \( s = S \), and the phase \( \xi(z) \) and the attenuation \( \xi(L) \) are given by

\[
\phi(z) = \int_0^z \kappa(z')dz' + \frac{k_0 n_a}{\cos \theta} + \frac{k_0}{n_2} \int_0^z I'(z')dz' + n_a \int_0^z z' I'(z')dz',
\]

\[
\xi(L) = \int_0^\infty \mu(z')dz' + \frac{\alpha L}{2 \cos \theta} + \frac{\beta}{2 \cos \theta} \int_0^\infty I'(z')dz'.
\]

Equation (5) can be solved for \( I \) together with Eqs. (6) and (7) for \( \phi(z) \) and \( \xi(L) \), respectively, as they both depend on \( I \) in the nonlinear regime. The resulting solution will be multivalued, indicating that the resonator will become bistable when the pump has a wavelength (or phase) detuned from the resonance and the input intensity \( |a|^2 \) is large enough. The solutions are described graphically in Fig. 2 by plotting the left-hand side (LHS) of Eq. (5), which is a straight line with tilt \( 1/|a|^2 \), and the right-hand side (RHS), which is the same curve as the cavity spectrum, with regard to \( I \). If the pump wavelength is at the resonant wavelength or has a small phase detuning \( \phi \) [if the first term of Eq. (6) at \( z=L \) is zero or \( \phi \)], then the solution is still single valued, and as the input intensity increases it increases following the resonant curve (see the circles in Fig. 2). When the pump wavelength has a large phase detuning \( \phi' \), as the input intensity increases, the solution is at first single valued and gradually increases. However, after the straight line crosses the peak of the resonant curve of the RHS, the solution becomes triple valued, and the physical solution is the smallest (largest) point for the ascending (descending) of the pump pulse. Finally, after the crossing point of the smallest solution vanishes, the solution jumps to the largest value and again becomes single valued. If we compare the solutions with no detuning to that with a value of \( \phi \) (see Fig. 2), the required input intensity \( |a|^2 \) for \( \phi \) detuning is smaller than that for no detuning, for the same modulation depth \( \delta \) and the same phase shift \( \phi \). Thus, with an appropriate phase detuning, the required input intensity can be significantly reduced. Solving Eq. (5), we can easily predict the optimum pump phase detuning and the required pump power for switching operation by using the material and the measured linear parameters of the device.

We first simplify integral equations (5)–(7) by removing the \( z \) dependence in \( I \). We assume that the phase and the loss inside the cavity are independent of \( z \) as \( \phi(z) = m \pi, \xi(z) = n \xi(L) \). With this assumption, we can use the position-independent intensity \( I_p \), which represents the peak intensity inside the cavity with the loss at the center of the cavity, instead of \( I(z) \) in Eqs. (5)–(7). Thus, we can simplify the integrals in Eqs. (6) and (7) as

\[
\phi(L) = \frac{k_0 n_1 L}{\cos \theta} + \frac{k_0 n_2 L_{\text{eff}1}}{\cos \theta} I_p + \frac{k_0 n_2 L_{\text{eff}2}}{\cos \theta} I_p^2,
\]

\[
\xi(L) = \frac{\alpha L}{2 \cos \theta} + \frac{\beta}{2 \cos \theta} I_p L_{\text{eff}1},
\]

where the effective length \( L_{\text{eff}1} \) and \( L_{\text{eff}2} \) are given by

\[
L_{\text{eff}1} = \int_0^L (z')dz'/I_p, \quad L_{\text{eff}2} = \int_0^L I'(z')dz'/I_p^2, \quad \beta = n_1 = n_2 = 0.
\]

Note that with an approximation of \( e^{-2-1} + x \) and \( \cos 2x - 2x^2 \), which is appropriate for a high-finesse and low-loss cavity mode, Eq. (5) reduces to a 6th-order algebraic equation for \( I_p \).

Next we compare our analysis with the experimental data from Ref. 4, where linear parameters of a PhC microcavity and the nonlinear operation were measured. We use the following measured parameters for the linear regime of operation: resonant wavelength \( \lambda_0 = 1547.68 \text{ nm} \) (\( \lambda_0 = \lambda_N \), the degenerate case), free spectrum range \( \text{FSR} = 1568.05 - 1530.47 = 37.58 \text{ nm} \), FHWM of the resonant mode of \( \Delta \lambda = 0.84 \text{ nm} \), waveguide loss \( \alpha (\text{dB}) = 1 \text{ dB/mm} \), cavity length \( L = 4 \times 0.42 = 1.68 \mu\text{m} \), and core area \( A = 0.2 \times 0.42 = 0.145 \mu\text{m}^2 \). We estimate the following values: (i) group index \( n_g = 15.2 \) from \( \lambda_0 \) and FSR and (ii) reflectance \( R = 0.9974 \) from \( \Delta \lambda \) and \( \alpha (\text{dB}) \). \( S \) is set to zero. We assumed material parameters \( n_1 = 3.46, n_2 = 0.45 \times 10^{-1} \text{ cm}^2/\text{W}, \) and \( \beta = 0.8 \text{ cm/GW} \). To predict the nonlinear operation, we first calculate \( n_d = -4.7 \times 10^{-30} \text{ m}^4/\text{W}^2 \) with \( \tau_p = 7.5 \text{ ps} \), which was used in the experiment in Ref. 4. Estimating the coeffi-
In conclusion, we derived an equation that describes the nonlinear operation of a Fabry–Perot resonator with a large group index waveguide. Specifically, a silicon PhC microcavity with nonlinearities due to the two-photon-excited free carrier effect and the Kerr effect was assumed. The equation clearly explained the bistability of the device and the reduction of the required pump energy for a specific nonlinear phase shift or modulation depth at an appropriate phase detuning from the resonance. We presented a simple procedure to predict the required optical pump energy for the modulation and the resulting modulation depth by using the equation and the device parameters. Compared with coupled mode analysis or rate equation analysis, this procedure makes it straightforward to see intuitively why bistable operation happens or how phase detuning and loss due to TPA affect the bistability and the device operation.

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References