

a transparent medium (with purely real permittivity). We obtained the Lorentzian in this manner in Section 5, and most authors who report estimates of Purcell factor use the same approach [4,8,11,14,32,33].

Yet even if we accept the assumption, embodied in Eq. (14), that an emitter interacts only with the electromagnetic field modes and does not directly interact with other emitters, these other emitters, just like the cavity boundary, can still modify its emission rate by altering the state of the field modes. The effect of emitters may be less important than that of the cavity boundary in gas lasers and in quantum dot lasers as long as the number of emitters remains small. In bulk and MQW semiconductor lasers, this is not so. Material loss in unpumped InGaAsP at the mode frequency may range from $3 \cdot 10^3 \text{cm}^{-1}$ to 10^4cm^{-1} , depending on the difference between the mode frequency and the material bandgap [44]. If this loss were included, the Q factor of the TE₀₁₂ mode in Fig. 2 would drop from the transparent medium value, 479, to as low as 16. The corresponding Lorentzian linewidth would become comparable to the width of the inhomogeneous broadening spectrum $D(\omega_{21})$.

While the modification of the modes, and hence of spontaneous emission rates, by the gain medium cannot be ignored, it is also unclear how it can be consistently included in the present model. Unlike the cavity boundary, an unpumped gain medium cannot be treated as a thermal reservoir. Recall that a reservoir must become completely disordered, i.e. a reservoir mode must cycle through all its possible states, over a time $\tau_{\text{reservoir}}$ that is short relative to the rate of change of the field mode to which it is coupled. For the unpumped gain medium, the dephasing time scale $\tau_{\text{reservoir}}$ would be on the order of $\tau_{\text{coll}} \sim 0.3 \text{ps}$. Yet the damping it inflicts on the mode is so severe that the mode decays in as little as 0.012ps (based on $Q = 16$ and $\omega = 1.330 \cdot 10^{15} \text{rad/s}$). Under such strong damping, even the treatment of the cavity boundary as a thermal reservoir becomes questionable, although the electron collision time in metals is on the order of 0.01ps [45,46].

It is also worth noting that once pumping is introduced, the (classically defined) Q factor of the mode rises and reaches a theoretical value of infinity at the lasing threshold. It seems likely that, in the quantum mechanical treatment, the manner in which the gain medium modifies the mode and, through the mode, modifies the emission rates, would also change as pumping is added. In this situation, however, the gain medium conforms even less to the thermal reservoir model, for it is no longer at equilibrium with the mode, and so Eq. (8) cannot be used. Furthermore, as pumping is increased and the field builds up in the cavity, the transition rate grows and may eventually exceed the collision rate, violating the emitter-mode weak coupling condition, $P_{2 \rightarrow 1}(t) \ll 1$. Under this circumstance, Eq. (9) can no longer be used. Intuitively, one may expect the gain material to partly compensate the dissipation at the cavity boundary, while at the same time contribute randomness to the mode state through spontaneous emission. Formal quantum mechanical treatment of the Purcell effect in semiconductor lasers, incorporating the effect of the gain medium, is the subject of future research.

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